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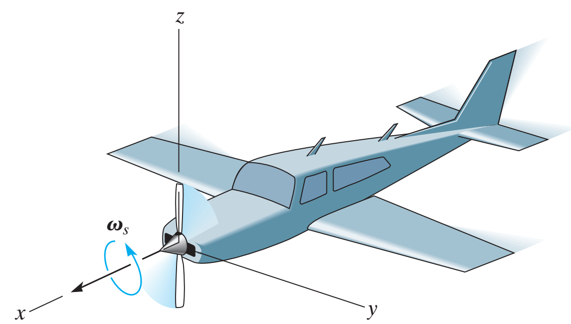
20–1.

The propeller of an airplane is rotating at a constant speed $\omega_s \mathbf{i}$, while the plane is undergoing a turn at a constant rate ω_t. Determine the angular acceleration of the propeller if (a) the turn is horizontal, i.e., $\omega_t \mathbf{k}$, and (b) the turn is vertical, downward, i.e., $\omega_t \mathbf{j}$.



SOLUTION

(a) For ω_s, $\Omega = \omega_t \mathbf{k}$.

$$(\dot{\omega}_s)_{XYZ} = (\dot{\omega}_s)_{xyz} + \Omega \times \omega_s$$

$$= \mathbf{0} + (\omega_t \mathbf{k}) \times (\omega_s \mathbf{i}) = \omega_s \omega_t \mathbf{j}$$

For ω_t, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_t)_{XYZ} = (\dot{\omega}_t)_{xyz} + \Omega \times \omega_t \mathbf{k} = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\alpha = \dot{\omega} = (\dot{\omega}_s)_{XYZ} + (\dot{\omega}_t)_{XYZ}$$

$$\alpha = \omega_s \omega_t \mathbf{j} + \mathbf{0} = \omega_s \omega_t \mathbf{j}$$ **Ans.**

(b) For ω_s, $\Omega = \omega_t \mathbf{j}$.

$$(\dot{\omega}_s)_{XYZ} = (\dot{\omega}_s)_{xyz} + \Omega \times \omega_s$$

$$= 0 + (\omega_t \mathbf{j}) \times (\omega_s \mathbf{i}) = -\omega_s \omega_t \mathbf{k}$$

For ω_t, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_t)_{XYZ} = (\dot{\omega}_t)_{xyz} + \Omega \times \omega_t = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\alpha = \dot{\omega} = (\dot{\omega}_s)_{XYZ} + (\dot{\omega}_t)_{XYZ}$$

$$\alpha = -\omega_s \omega_t \mathbf{k} + \mathbf{0} = -\omega_s \omega_t \mathbf{k}$$ **Ans.**

Ans:
(a) $\boldsymbol{\alpha} = \omega_s \omega_t \mathbf{j}$
(b) $\boldsymbol{\alpha} = -\omega_s \omega_t \mathbf{k}$

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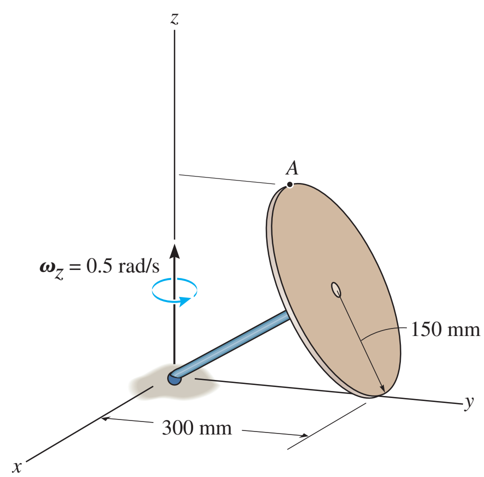
20–2.

The disk rotates about the z axis at a constant rate $\omega_z = 0.5$ rad/s without slipping on the horizontal plane. Determine the velocity and the acceleration of point A on the disk.



SOLUTION

Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i**, **j**, **k** components. Since the disk rolls without slipping, then its angular velocity $\omega = \omega_s + \omega_z$ is always directed along the instantaneuos axis of zero velocity (y axis). Thus,

$$\omega = \omega_s + \omega_z$$

$$-\omega\mathbf{j} = -\omega_s \cos 30°\mathbf{j} - \omega_s \sin 30°\mathbf{k} + 0.5\mathbf{k}$$

Equating **k** and **j** components, we have

$$0 = -\omega_s \sin 30° + 0.5 \qquad \omega_s = 1.00 \text{ rad/s}$$

$$-\omega = -1.00 \cos 30° \qquad \omega = 0.8660 \text{ rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating the time rate of change of *angular velocity* with respect to the fixed XYZ frame. Since ω always lies in the fixed X–Y plane, then $\omega = \{-0.8660\mathbf{j}\}$ rad/s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{0.5\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega})_{xyz} = \mathbf{0}$, we have

$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega = \mathbf{0} + 0.5\mathbf{k} \times (-0.8660\mathbf{j}) = \{0.4330\mathbf{i}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{(0.3 - 0.3\cos 60°)\mathbf{j} + 0.3\sin 60°\mathbf{k}\}$ m $= \{0.15\mathbf{j} + 0.2598\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k}) = \{-0.225\mathbf{i}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (0.4330\mathbf{i}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})$$

$$+ (-0.8660\mathbf{j}) \times [(-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})]$$

$$= \{-0.1125\mathbf{j} - 0.130\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-0.225\mathbf{i}\}$ m/s
$\mathbf{a}_A = \{-0.1125\mathbf{j} - 0.130\mathbf{k}\}$ m/s^2

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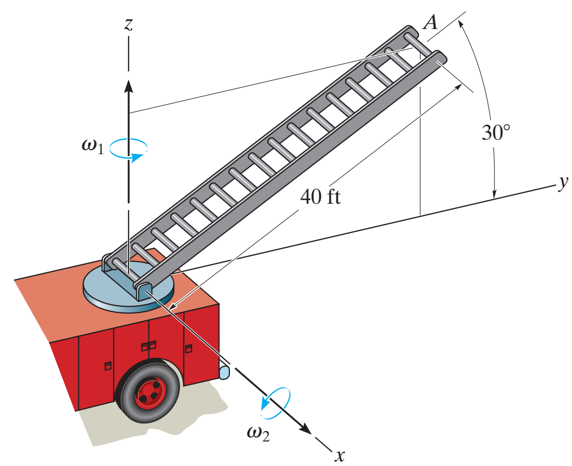
20–3.

The ladder of the fire truck rotates around the z axis with an angular velocity $\omega_1 = 0.15$ rad/s, which is increasing at 0.8 rad/s^2. At the same instant it is rotating upward at a constant rate $\omega_2 = 0.6$ rad/s. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



SOLUTION

$\omega = \omega_1 + \omega_2 = 0.15\mathbf{k} + 0.6\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

Angular acceleration: For ω_1, $\omega = \omega_1 = \{0.15\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \omega \times \omega_2$$

$$= 0 + (0.15\mathbf{k}) \times (0.6\mathbf{i}) = \{0.09\mathbf{j}\} \text{ rad/s}^2$$

For ω_1, $\Omega = 0$.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \omega \times \omega_1 = (0.8\mathbf{k}) + 0 = \{0.8\mathbf{k}\} \text{ rad/s}^2$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = 0.8\mathbf{k} + 0.09\mathbf{j} = \{0.09\mathbf{j} + 0.8\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = 40 \cos 30°\mathbf{j} + 40 \sin 30°\mathbf{k} = \{34.641\mathbf{j} + 20\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$$

$$= \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\} \text{ ft/s}$$ **Ans.**

$$\mathbf{a}_A = \alpha \times \mathbf{r} + \omega \times \mathbf{v}_A$$

$$= (0.09\mathbf{j} + 0.8\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k}) + (0.6\mathbf{i} + 0.15\mathbf{k}) \times (-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k})$$

$$= \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\}$ ft/s^2

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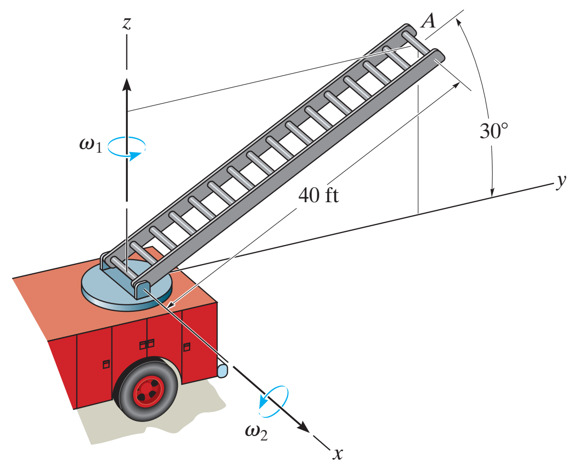
***20–4.**

The ladder of the fire truck rotates around the z axis with an angular velocity of $\omega_1 = 0.15$ rad s, which is increasing at 0.2 rad/s^2. At the same instant it is rotating upward at $\omega_2 = 0.6$ rad/s while increasing at 0.4 rad/s^2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



SOLUTION

$\mathbf{r}_{A/O} = 40 \cos 30°\mathbf{j} + 40 \sin 30°\mathbf{k}$

$\mathbf{r}_{A/O} = \{34.641\mathbf{j} + 20\mathbf{k}\}$ ft

$\Omega = \omega_1 \mathbf{k} + \omega_2 \mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

$\dot{\omega} = \dot{\omega}_1 \mathbf{k} + \dot{\omega}_2 \mathbf{i} + \omega_1 \mathbf{k} \times \omega_2 \mathbf{i}$

$\dot{\Omega} = 0.2\mathbf{k} + 0.4\mathbf{i} + 0.15\mathbf{k} \times 0.6\mathbf{i} = \{0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_A = \Omega \times \mathbf{r}_{A/O} = (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s **Ans.**

$\mathbf{a}_A = \Omega \times (\Omega \times \mathbf{r}_{A/O}) + \dot{\omega} \times \mathbf{r}_{A/O}$

$\mathbf{a}_A = (0.6\mathbf{i} + 0.15\mathbf{k}) \times [(0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})]$

$\quad + (0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}$ ft/s^2

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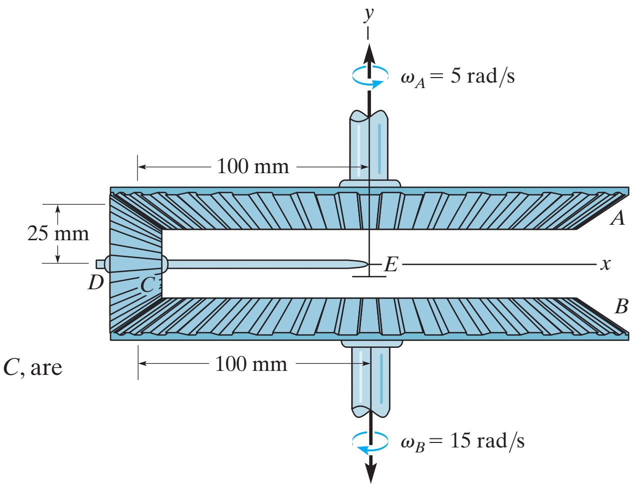
20–5.

If the plate gears A and B are rotating with the angular velocities shown, determine the angular velocity of gear C about the shaft DE. What is the angular velocity of DE about the y axis?



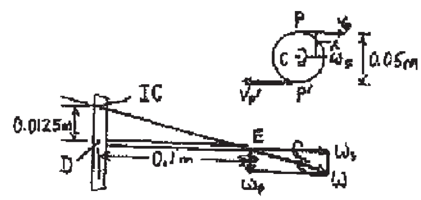
SOLUTION

The speeds of points P and P', located at the top and bottom of gear C, are

$$v_p = (5)(0.1) = 0.5 \text{ m/s}$$

$$v_{p'} = (15)(0.1) = 1.5 \text{ m/s}$$

The IC is located as shown.



$$\frac{0.5}{x} = \frac{1.5}{(0.05 - x)}; \quad x = 0.0125 \text{ m}$$

$$\frac{\omega_s}{0.1} = \frac{\omega_p}{0.0125}; \quad \omega_s = 8\,\omega_p$$

$$\omega = \omega_s \mathbf{i} - \omega_p \mathbf{j} = \omega_s \mathbf{i} - \frac{1}{8}\omega_p \mathbf{j}$$

$$\mathbf{v} = \omega \times \mathbf{r}$$

$$0.5\mathbf{k} = \left(\omega_p \mathbf{i} - \frac{1}{8}\omega_s \mathbf{j}\right) \times (-0.1\mathbf{i} + 0.025\mathbf{j})$$

$$0.5\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_s & -\frac{1}{8}\omega_s & 0 \\ -0.1 & 0.025 & 0 \end{vmatrix} = 0.0125\omega_s \mathbf{k}$$

$$\omega_s = \frac{0.5}{0.0125} = 40 \text{ rad/s}$$ **Ans.** (Angular velocity of C about DE)

$$\omega_p = \frac{1}{8}(40) = 5 \text{ rad/s}$$ **Ans.** (Angular velocity of DE about y axis)

Ans:
$(\omega_C)_{DE} = 40 \text{ rad/s}$
$(\omega_{DE})_y = 5 \text{ rad/s}$

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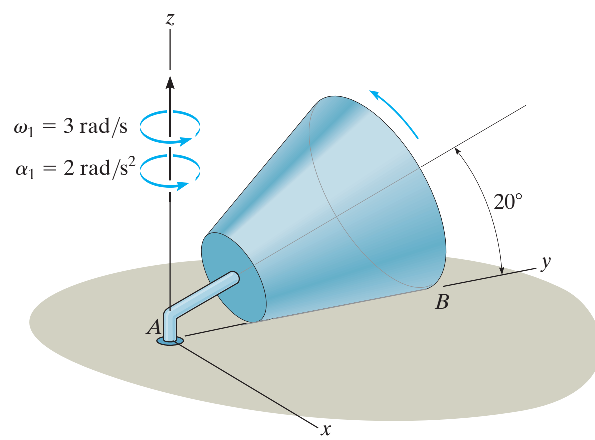
20–6.

The conical spool rolls on the plane without slipping. If the axle has an angular velocity of $\omega_1 = 3$ rad/s and an angular acceleration of $\alpha_1 = 2$ rad/s^2 at the instant shown, determine the angular velocity and angular acceleration of the spool at this instant.



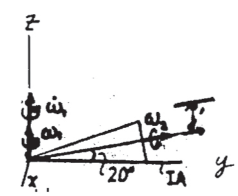
SOLUTION

$\omega_1 = 3$ rad/s

$$\omega_2 = -\frac{3}{\sin 20°} = -8.7714 \text{ rad/s}$$

$\omega = \omega_1 + \omega_2 = 3\mathbf{k} - 8.7714 \cos 20°\mathbf{j} - 8.7714 \sin 20°\mathbf{k}$

$= \{-8.24\mathbf{j}\}$ rad/s **Ans.**



$(\dot{\omega}_1)_{xyz} = 2$ rad/s^2

$$(\dot{\omega}_2)_{xyz} = -\frac{2}{\sin 20°} = -5.8476 \text{ rad/s}^2$$

$\alpha = \dot{\omega} = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 + (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2$

$= 2\mathbf{k} + \mathbf{0} + (-5.8476 \cos 20°\mathbf{j} - 5.8476 \sin 20°\mathbf{k}) + (3\mathbf{k}) \times (-8.7714 \cos 20°\mathbf{j} - 8.7714 \sin 20°\mathbf{k})$

$\alpha = \{24.7\mathbf{i} - 5.49\mathbf{j}\}$ rad/s^2 **Ans.**

Ans:
$\boldsymbol{\omega} = \{-8.24\mathbf{j}\}$ rad/s
$\boldsymbol{\alpha} = \{24.7\mathbf{i} - 5.49\mathbf{j}\}$ rad/s^2

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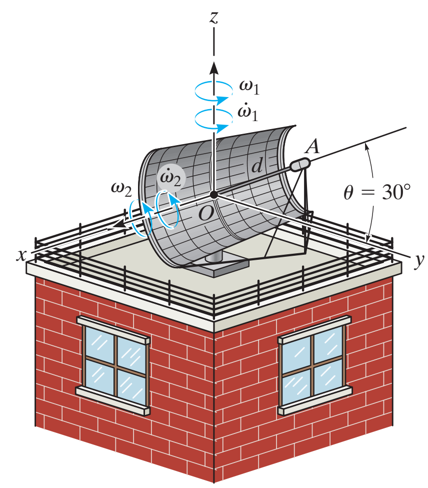
20–7.

At a given instant, the antenna has an angular motion $\omega_1 = 3$ rad/s and $\dot{\omega}_1 = 2$ rad/s^2 about the z axis. At this same instant $\theta = 30°$, the angular motion about the x axis is $\omega_2 = 1.5$ rad/s, and $\dot{\omega}_2 = 4$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant. The distance from O to A is $d = 3$ ft.



SOLUTION

$$\mathbf{r}_A = 3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k} = \{2.598\mathbf{j} + 1.5\mathbf{k}\}\text{ ft}$$

$$\Omega = \omega_1 + \omega_2 = 3\mathbf{k} + 1.5\mathbf{i}$$

$$\mathbf{v}_A = \Omega \times \mathbf{r}_A$$

$$\mathbf{v}_A = (3\mathbf{k} + 1.5\mathbf{i}) \times (2.598\mathbf{j} + 1.5\mathbf{k})$$

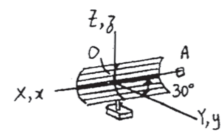
$$= -7.794\mathbf{i} + 3.897\mathbf{k} - 2.25\mathbf{j}$$

$$= \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\}\text{ ft/s} \qquad \textbf{Ans.}$$

$$\dot{\Omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$= (2\mathbf{k} + 0) + (4\mathbf{i} + 3\mathbf{k} \times 1.5\mathbf{i})$$

$$= 4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}$$



$$\mathbf{a}_A = \dot{\omega} \times r_A + \Omega \times \mathbf{v}_A$$

$$\mathbf{a}_A = (4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}) \times (2.598\mathbf{j} + 1.5\mathbf{k}) + (3\mathbf{k} + 1.5\mathbf{i}) \times (-7.794\mathbf{i} - 2.25\mathbf{j} + 3.879\mathbf{k})$$

$$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\}$ ft/s^2

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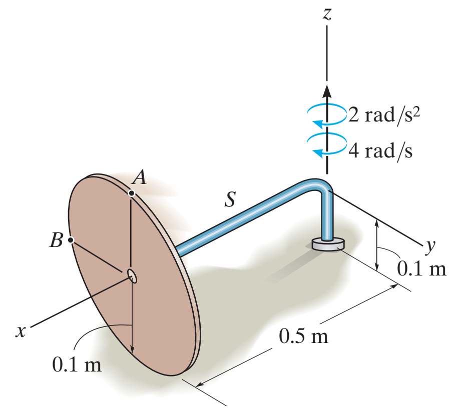
***20–8.**

The disk rotates about the shaft S, while the shaft is turning about the z axis at a rate of $\omega_z = 4$ rad/s, which is increasing at 2 rad/s^2. Determine the velocity and acceleration of point A on the disk at the instant shown. No slipping occurs.



SOLUTION

Angular Velocity. The instantaneous axis of zero velocity (IA) is indicated in Fig. a, Here, the resultant angular velocity is always directed along IA. The fixed XYZ reference frame is set to coincide with the rotating xyz frame.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \omega_2$$

$$\frac{5}{\sqrt{26}}\omega\mathbf{i} - \frac{1}{\sqrt{26}}\omega\mathbf{k} = -4\mathbf{k} + \omega_2\mathbf{i}$$

Equating **k** and **i** components,

$$-\frac{1}{\sqrt{26}}\omega = -4 \qquad \omega = 4\sqrt{26}\text{ rad/s}$$

$$\frac{5}{\sqrt{26}}\left(4\sqrt{26}\right) = \omega_2 \quad \omega_2 = 20\text{ rad/s}$$

Thus, $\boldsymbol{\omega} = \dfrac{5}{\sqrt{26}}(4\sqrt{26})\mathbf{i} - \dfrac{1}{\sqrt{26}}(4\sqrt{26})\mathbf{k} = \{20\mathbf{i} - 4\mathbf{k}\}$ rad/s

Angular Acceleration. The direction of $\boldsymbol{\omega}_2$ does not change with reference to xyz rotating frame if this frame rotates with $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = \{-4\mathbf{k}\}$ rad/s. Here

$$\frac{(\dot{\omega}_2)_{xyz}}{(\dot{\omega}_1)_{xyz}} = \frac{5}{1}; \quad (\dot{\omega}_2)_{xyz} = 5(\dot{\omega}_1)_{xyz} = 5(2) = 10\text{ rad/s}^2$$

Therefore

$$\begin{aligned}\dot{\boldsymbol{\omega}}_2 &= (\dot{\omega}_2)_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}_2 \\ &= 10\mathbf{i} + (-4\mathbf{k}) \times (20\mathbf{i}) \\ &= \{10\mathbf{i} - 80\mathbf{j}\}\text{ rad/s}^2\end{aligned}$$

Since the direction of $\boldsymbol{\omega}_1$ will not change that is always along z axis when $\boldsymbol{\Omega} = \boldsymbol{\omega}_1$, then

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\boldsymbol{\omega}}_1)_{xyz} + \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_1$$

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\omega}_1)_{xyz} = \{-2\mathbf{k}\}\text{ rad/s}^2$$

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***20–8. Continued**

Finally,

$$\boldsymbol{\alpha} = \dot{\omega}_1 + \dot{\omega}_2$$

$$= -2\mathbf{k} + 10\mathbf{i} - 80\mathbf{j}$$

$$= \{10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration. Here $r_A = \{0.5\mathbf{i} + 0.1\mathbf{k}\}$ m

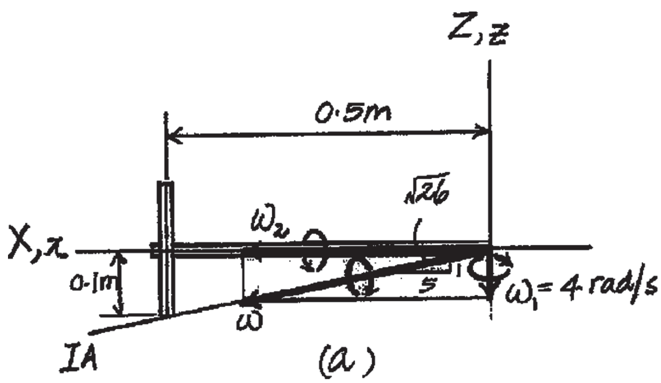
$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A = (20\mathbf{i} - 4\mathbf{k}) \times (0.5\mathbf{i} + 0.1\mathbf{k})$$

$$= \{-4.00\mathbf{j}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times r_A)$$

$$= (10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}) \times (0.5\mathbf{i} + 0.1\mathbf{k}) + (20\mathbf{i} - 4\mathbf{k}) \times (-4.00\mathbf{j})$$

$$= \{-24\mathbf{i} - 2\mathbf{j} - 40\mathbf{k}\} \text{ m/s}^2$$ **Ans.**



Ans:
$\mathbf{v}_A = \{-4.00\mathbf{j}\}$ m/s
$\mathbf{a}_A = \{-24\mathbf{i} - 2\mathbf{j} - 40\mathbf{k}\}$ m/s^2

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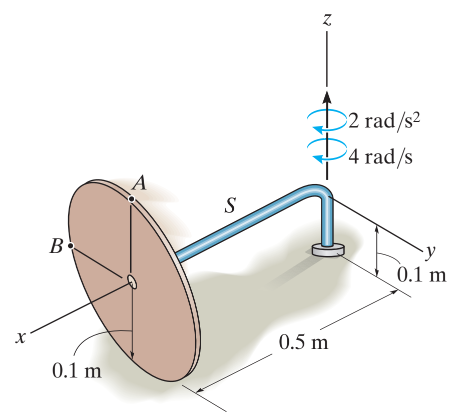
20–9.

The disk rotates about the shaft S, while the shaft is turning about the z axis at a rate of $\omega_z = 4$ rad/s, which is increasing at 2 rad/s². Determine the velocity and acceleration of point B on the disk at the instant shown. No slipping occurs.



SOLUTION

Angular velocity. The instantaneous axis of zero velocity (IA) is indicated in Fig. a. Here the resultant angular velocity is always directed along IA. The fixed XYZ reference frame is set coincide with the rotating xyz frame.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$

$$\frac{5}{\sqrt{26}}\omega\mathbf{i} - \frac{1}{\sqrt{26}}\omega\mathbf{k} = -4\mathbf{k} + \omega_2\mathbf{i}$$

Equating **k** and **i** components,

$$-\frac{1}{\sqrt{26}}\omega = -4 \qquad \omega = 4\sqrt{26}\text{ rad/s}$$

$$\frac{5}{\sqrt{26}}\left(4\sqrt{26}\right) = \omega_2 \quad \omega_2 = 20\text{ rad/s}$$

Thus, $\omega = \frac{5}{\sqrt{26}}\left(4\sqrt{26}\right)\mathbf{i} - \frac{1}{\sqrt{26}}\left(4\sqrt{26}\right)\mathbf{k} = \{20\mathbf{i} - 4\mathbf{k}\}$ rad/s

Angular Acceleration. The direction of $\boldsymbol{\omega}_2$ does not change with reference to xyz rotating frame if this frame rotates with $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = \{-4\mathbf{k}\}$ rad/s. Here

$$\frac{(\dot{\omega}_2)_{xyz}}{(\dot{\omega}_1)_{xyz}} = \frac{5}{1}; \quad (\dot{\omega}_2)_{xyz} = 5(\dot{\omega}_1)_{xyz} = 5(2) = 10\text{ rad/s}^2$$

Therefore,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}_2$$

$$= 10\mathbf{i} + (-4\mathbf{k}) \times (20\mathbf{i})$$

$$= \{10\mathbf{i} - 80\mathbf{j}\}\text{ rad/s}^2$$

Since the direction of $\boldsymbol{\omega}_1$ will not change that is always along z axis when $\boldsymbol{\Omega} = \boldsymbol{\omega}_1$, then

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\omega}_1)_{xyz} + \boldsymbol{\omega}_1 \times \omega_1$$

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\omega}_1)_{xyz} = \{-2\mathbf{k}\}\text{ rad/s}^2$$

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20–9. Continued

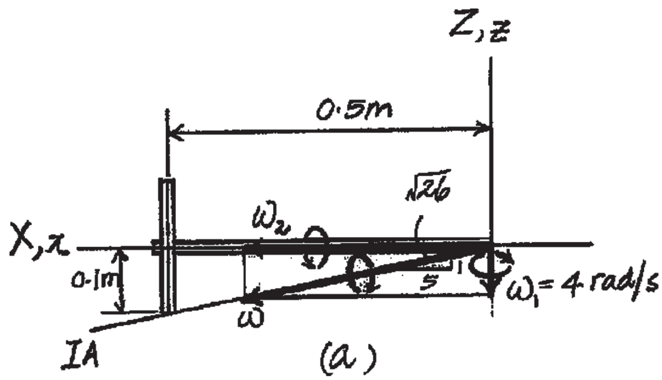
Finally,

$$\alpha = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2$$
$$= -2\mathbf{k} + (10\mathbf{i} - 80\mathbf{j})$$
$$= \{10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}\}\ \text{rad/s}^2$$

Velocity and Acceleration. Here $r_B = \{0.5\mathbf{i} - 0.1\mathbf{j}\}$ m

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B = (20\mathbf{i} - 4\mathbf{k}) \times (0.5\mathbf{i} - 0.1\mathbf{j})$$
$$= \{-0.4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_B + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_B)$$
$$= (10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}) \times (0.5\mathbf{i} - 0.1\mathbf{j}) + (20\mathbf{i} - 4\mathbf{k}) \times (-0.4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$
$$= \{-8.20\mathbf{i} + 40.6\mathbf{j} - \mathbf{k}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



Ans:
$\mathbf{v}_B = \{-0.4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{-8.20\mathbf{i} + 40.6\mathbf{j} - \mathbf{k}\}$ rad/s^2

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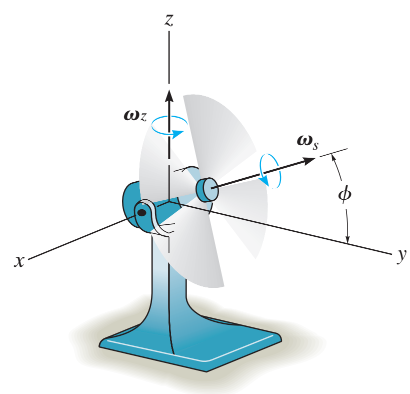
20–10.

The electric fan is mounted on a swivel support such that the fan rotates about the z axis at a constant rate of $\omega_z = 1$ rad/s and the fan blade is spinning at a constant rate $\omega_s = 60$ rad/s. If $\phi = 45°$ for the motion, determine the angular velocity and the angular acceleration of the blade.



SOLUTION

$\omega = \omega_z + \omega_s$

$= 1\mathbf{k} + 60\cos 45°\mathbf{j} + 60\sin 45°\mathbf{k}$

$= 42.426\mathbf{j} + 43.426\mathbf{k}$

$= \{42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s **Ans.**

$\dot{\omega} = \dot{\omega}_z + \dot{\omega}_s$

$= 0 + 0 + \omega_z \times \omega_s$

$= 1\mathbf{k} \times 42.426\mathbf{j} + 43.426\mathbf{k}$ **Ans.**

$= \{-42.4\mathbf{i}\}$ rad/s^2 **Ans.**

Ans:
$\boldsymbol{\omega} = \{42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s
$\boldsymbol{\alpha} = \{-42.4\mathbf{i}\}$ rad/s^2

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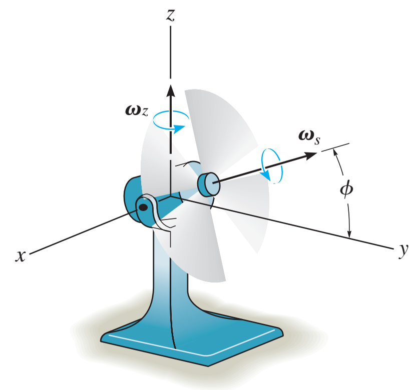
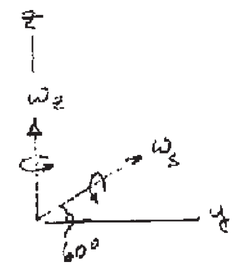
20–11.

The electric fan is mounted on a swivel support such that the fan rotates about the z axis at a constant rate of $\omega_z = 1$ rad/s and the fan blade is spinning at a constant rate $\omega_s = 60$ rad/s. If at the instant $\phi = 45°$, $\dot{\phi} = 2$ rad/s for the motion, determine the angular velocity and the angular acceleration of the blade.



SOLUTION



$\omega = \omega_z + \omega_s + \omega_x$

$= 1\mathbf{k} + 60\cos 45°\mathbf{j} + 60\sin 45°\mathbf{k} + 2\mathbf{i}$

$= 2\mathbf{i} + 42.426\mathbf{j} + 43.426\mathbf{k}$

$= \{2\mathbf{i} + 42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s **Ans.**

$\dot{\omega} = \dot{\omega}_z + \dot{\omega}_s + \dot{\omega}_x$

$= 0 + (\omega_z + \omega_x) \times \omega_s + \omega_z \times \omega_x$

$= 0 + (1\mathbf{k} + 2\mathbf{i}) \times (42.426\mathbf{j} + 43.426\mathbf{k}) + 1\mathbf{k} \times (2\mathbf{i})$

$= -42.426\mathbf{i} + 84.853\mathbf{k} - 84.853\mathbf{j} + 2\mathbf{j}$

$= \{-42.4\mathbf{i} - 82.9\mathbf{j} + 84.9\mathbf{k}\}$ rad/s^2 **Ans.**

Ans:
$\boldsymbol{\omega} = \{2\mathbf{i} + 42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s
$\boldsymbol{\alpha} = \{-42.4\mathbf{i} - 82.9\mathbf{j} + 84.9\mathbf{k}\}$ rad/s^2

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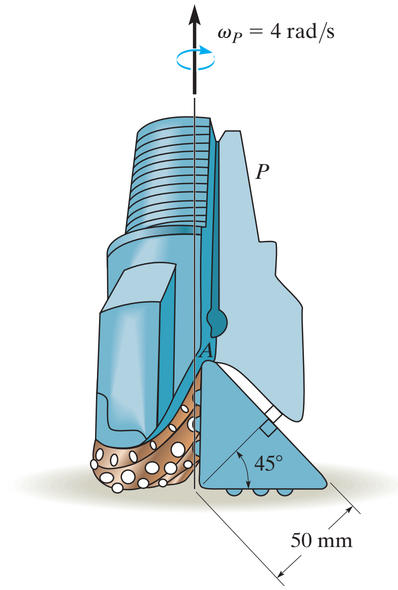
***20–12.**

The drill pipe P turns at a constant angular rate $\omega_P = 4$ rad/s. Determine the angular velocity and angular acceleration of the conical rock bit, which rolls without slipping. Also, what are the velocity and acceleration of point A?



SOLUTION

$\omega = \omega_1 + \omega_2$

Since ω acts along the instantaneous axis of zero velocity,

$\omega\mathbf{j} = \omega_1\mathbf{k} + \omega_2 \cos 45°\mathbf{j} + \omega_2 \sin 45°\mathbf{k}$

Setting $\omega_1 = 4$ rad/s

$\omega\mathbf{j} = 4\mathbf{k} + 0.707\omega_2\mathbf{j} + 0.707\omega_2\mathbf{k}$

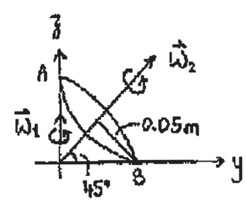
Equating components

$\omega = 0.707\omega_2$

$0 = 4 + 0.707\omega_2$

$\omega = -4$ rad/s

$\omega_2 = -5.66$ rad/s



Thus,

$\omega = \{-4.00\mathbf{j}\}$ rad/s **Ans.**

$\Omega = \omega_1$

$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$

$= 0 + \omega_1 \times (\omega)$

$= 0 + (4\mathbf{k}) \times (-4\mathbf{j})$

$\alpha = \dot{\omega} = \{16.01\}$ rad/s^2 **Ans.**

$\mathbf{v}_A = \omega \times \mathbf{r}_A$

$= (-4\mathbf{j}) \times [100(0.707)\mathbf{k}]$

$= \{-282.81\}$ mm/s

$\mathbf{v}_A = \{-0.283\mathbf{i}\}$ m/s **Ans.**

$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$

$= (16\mathbf{i}) \times (100)(0.707)\mathbf{k} + (-4\mathbf{j}) \times (-282.8\mathbf{i})$

$= \{-1131.2\mathbf{j} - 1131.2\mathbf{k}\}$ mm/s^2

$\mathbf{a}_A = \{-1.13\mathbf{j} - 1.13\mathbf{k}\}$ m/s^2 **Ans.**

Ans:
$\omega = \{-4.00\mathbf{j}\}$ rad/s
$\alpha = \{16.01\}$ rad/s^2
$\mathbf{v}_A = \{-0.283\mathbf{i}\}$ m/s
$\mathbf{a}_A = \{-1.13\mathbf{j} - 1.13\mathbf{k}\}$ m/s^2

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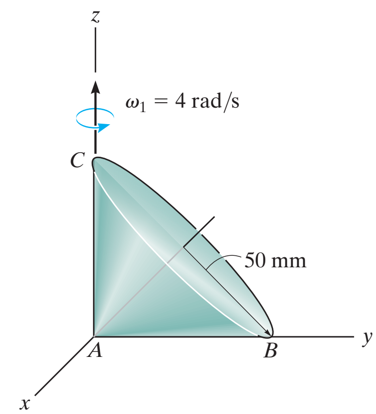
20–13.

The right circular cone rotates about the z axis at a constant rate of $\omega_1 = 4$ rad/s without slipping on the horizontal plane. Determine the magnitudes of the velocity and acceleration of points B and C.



SOLUTION

$\omega = \omega_1 + \omega_2$

Since ω acts along the instantaneous axis of zero velocity

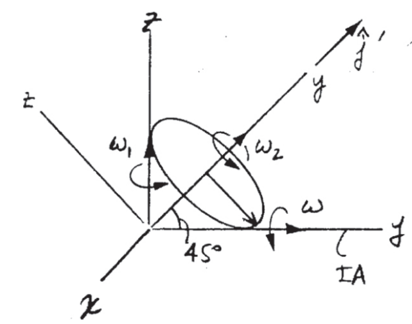
$\omega\mathbf{j} = 4\mathbf{k} + \omega_2 \cos 45°\,\mathbf{j} + \omega_2 \sin 45°\mathbf{k}.$

Equating components,

$\omega = 0.707\,\omega_2$

$0 = 4 + 0.707\,\omega_2$

$\omega = -4$ rad/s, $\quad \omega_2 = -5.66$ rad/s



Thus,

$\omega = \{-4\mathbf{j}\}$ rad/s

$\Omega = \omega_1$

$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \mathbf{0} + \omega_1 \times \omega_2$

$\quad = \mathbf{0} + (4\mathbf{k}) \times (-5.66 \cos 45°\mathbf{j} - 5.66 \sin 45°\mathbf{k})$

$\alpha = \dot{\omega} = \{16\mathbf{i}\}$ rad/s^2

$\mathbf{v}_B = \omega \times \mathbf{r}_B = (-4\mathbf{j}) \times (0.1(0.707)\mathbf{j}) = \mathbf{0}$

$v_B = 0$ **Ans.**

$\mathbf{v}_C = \omega \times \mathbf{r}_C = (-4\mathbf{j}) \times (0.1(0.707)\mathbf{k}) = \{-0.2828\mathbf{i}\}$ m/s

$v_C = 0.283$ m/s **Ans.**

$\mathbf{a}_B = \alpha \times \mathbf{r}_B + \omega \times \mathbf{v}_B = 16\mathbf{i} \times (0.1)(0.707)\mathbf{j} + \mathbf{0}$

$\mathbf{a}_B = \{1.131\mathbf{k}\}$ m/s

$a_B = 1.13$ m/s^2 **Ans.**

$\mathbf{a}_C = \alpha \times \mathbf{r}_C + \omega \times \mathbf{v}_C = 16\mathbf{i} \times (0.1)(0.707)\mathbf{k} + (-4\mathbf{j}) \times (-0.2828\mathbf{i})$

$\mathbf{a}_C = \{-1.131\mathbf{j} - 1.131\mathbf{k}\}$ m/s^2

$a_C = 1.60$ m/s^2 **Ans.**

Ans:
$v_B = 0$
$v_C = 0.283$ m/s
$a_B = 1.13$ m/s^2
$a_C = 1.60$ m/s^2

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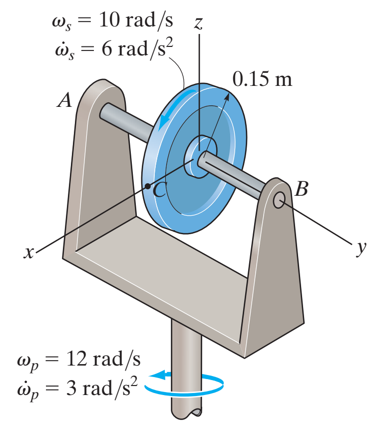
20–14.

The wheel is spinning about shaft AB with an angular velocity of $\omega_s = 10$ rad/s, which is increasing at a constant rate of $\dot{\omega}_s = 6\ \text{rad/s}^2$, while the frame precesses about the z axis with an angular velocity of $\omega_p = 12$ rad/s, which is increasing at a constant rate of $\omega_p = 3\ \text{rad/s}^2$. Determine the velocity and acceleration of point C located on the rim of the wheel at this instant.



SOLUTION

The XYZ fixed reference frame is set to coincide with the rotating xyz reference frame at the instant considered. Thus, the angular velocity of the wheel at this instant can be obtained by vector addition of ω_s and ω_p.

$$\omega = \omega_s + \omega_p = [10\mathbf{j} + 12\mathbf{k}]\ \text{rad/s}$$

The angular acceleration of the disk is determined from

$$\alpha = \dot{\omega} = \dot{\omega}_s + \dot{\omega}_p$$

If we set the xyz rotating frame to have an angular velocity of $\Omega = \omega_p = [12\mathbf{k}]$ rad/s, the direction of ω_s will remain unchanged with respect to the xyz rotating frame which is along the y axis. Thus,

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_p \times \omega_s = 6\mathbf{j} + (12\mathbf{k}) \times (10\mathbf{j}) = [-120\mathbf{i} + \mathbf{6j}]\ \text{rad/s}^2$$

Since ω_p is always directed along the Z axis where $\Omega = \omega_p$, then

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = 3\mathbf{k} + \mathbf{0} = [3\mathbf{k}]\ \text{rad/s}^2$$

Thus, $\alpha = (-120\mathbf{i} + 6\mathbf{j}) + 3\mathbf{k} = [-120\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}]\ \text{rad/s}^2$

Here, $\mathbf{r}_C = [0.15\mathbf{i}]$ m, so that

$$v_C = \omega \times \mathbf{r}_C = (10\mathbf{j} + 12\mathbf{k}) \times (0.15\mathbf{i}) = [1.8\mathbf{j} - 1.5\mathbf{k}]\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$a_C = \alpha \times \mathbf{r}_C + \omega \times (\omega \times \mathbf{r}_C)$$

$$= (-120\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \times (0.15\mathbf{i}) + (10\mathbf{j} + 12\mathbf{k}) \times [(10\mathbf{j} + 12\mathbf{k}) \times (0.15\mathbf{i})]$$

$$= [-36.6\mathbf{i} + 0.45\mathbf{j} - 0.9\mathbf{k}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{1.8\mathbf{j} - 1.5\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-36.6\mathbf{i} + 0.45\mathbf{j} - 0.9\mathbf{k}\}\ \text{m/s}^2$

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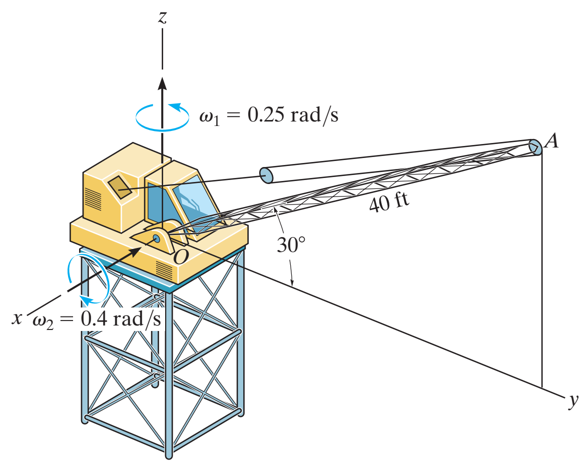
20–15.

At the instant shown, the tower crane rotates about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA rotates downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at this instant.



SOLUTION

$$\omega = \omega_1 + \omega_2 = \{-0.4\,\mathbf{i} + 0.25\mathbf{k}\} \text{ rad/s}$$

$$\Omega = \{0.25\,\mathbf{k}\} \text{ rad/s}$$

$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega = (-0.8\,\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\,\mathbf{i} + 0.25\mathbf{k})$$

$$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = 40 \cos 30°\mathbf{j} + 40 \sin 30°\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (1 - 0.4\,\mathbf{i} + 0.25\,\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$$

$$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\} \text{ ft/s}$$ **Ans.**

$$\mathbf{a}_A = \alpha \cdot \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$$

$$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ ft/s^2

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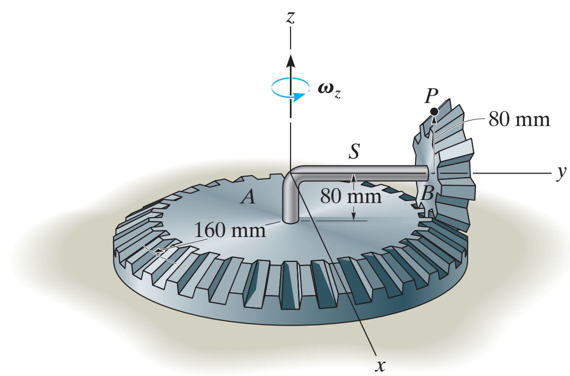
***20–16.**

Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 5$ rad/s, while increasing at 2 rad/s^2, determine the velocity and acceleration of point P at the instant shown. The face of gear B lies in a vertical plane.



SOLUTION

$\Omega = \{5\mathbf{k} - 10\mathbf{j}\}$ rad/s

$\dot{\Omega} = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_P = \Omega \times \mathbf{r}_P$

$\mathbf{v}_P \quad (5\mathbf{k} - 10\mathbf{j} \times (160\mathbf{j} + 80\mathbf{k})$

$\mathbf{v}_P = \{-1600\mathbf{i}\}$ mm/s

$= \{-1.60\mathbf{i}\}$ m/s **Ans.**

$\mathbf{a}_P = \Omega \times \mathbf{v}_P + \dot{\Omega} \times \mathbf{r}_P$

$\mathbf{a}_P = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\} \times (160\mathbf{j} + 80\mathbf{k}) + (-10\mathbf{j} + 5\mathbf{k}) \times (-1600\mathbf{i})$

$\mathbf{a}_P = \{-640\mathbf{i} - 12000\mathbf{j} - 8000\mathbf{k}\}$ mm/s^2

$\mathbf{a}_P = \{-0.640\mathbf{i} - 12.0\mathbf{j} - 8.00\mathbf{k}\}$ m/s^2 **Ans.**

Ans:
$\mathbf{v}_P = \{-1.60\mathbf{i}\}$ m/s
$\mathbf{a}_P = \{-0.640\mathbf{i} - 12.0\mathbf{j} - 8.00\mathbf{k}\}$ m/s^2

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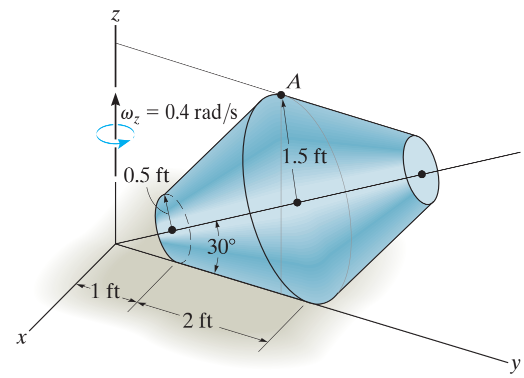
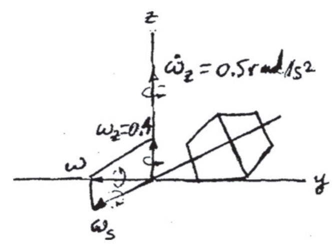
20–17.

The truncated double cone rotates about the z axis at $\omega_z = 0.4$ rad/s without slipping on the horizontal plane. If at this same instant ω_z is increasing at $\dot{\omega}_z = 0.5$ rad/s^2, determine the velocity and acceleration of point A on the cone.



SOLUTION



$$\theta = \sin^{-1}\left(\frac{0.5}{1}\right) = 30^\circ$$

$$\omega_s = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ rad/s}$$

$$\omega = 0.8 \cos 30^\circ = 0.6928 \text{ rad/s}$$

$$\omega = \{-0.6928\mathbf{j}\} \text{ rad/s}$$

$$\Omega = 0.4\mathbf{k}$$

$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega \qquad \textbf{(1)}$$

$$= 0.5\mathbf{k} + (0.4\mathbf{k}) \times (-0.6928\mathbf{j})$$

$$\dot{\omega} = 0.2771\mathbf{i} + 0.5\mathbf{k}$$

$$\mathbf{r}_A = (3 - 3\sin 30^\circ)\mathbf{j} + 3\cos 30^\circ\mathbf{k}$$

$$= (1.5\mathbf{j} + 2.598\mathbf{k}) \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (-0.6928\mathbf{j}) \times (1.5\mathbf{j} + 2.598\mathbf{k})$$

$$\mathbf{v}_A = \{-1.80\mathbf{i}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$$

$$= (0.2771\mathbf{i} + 0.5\mathbf{k}) \times (1.5\mathbf{j} + 2.598\mathbf{k}) + (-0.6928\mathbf{j}) \times (-1.80\mathbf{i})$$

$$\mathbf{a}_A = \{-0.750\mathbf{i} - 0.720\mathbf{j} - 0.831\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-1.80\mathbf{i}\}$ ft/s
$\mathbf{a}_A = \{-0.750\mathbf{i} - 0.720\mathbf{j} - 0.831\mathbf{k}\}$ ft/s^2

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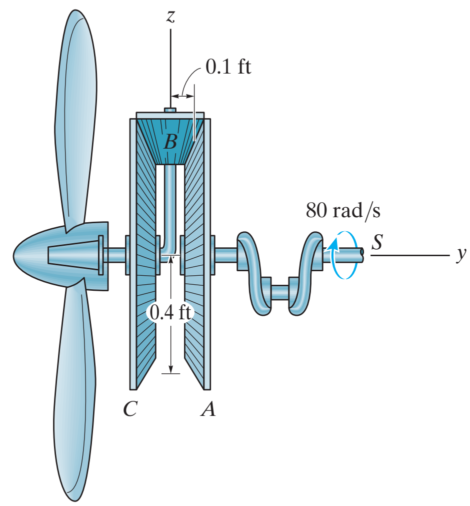
20–18.

Gear A is fixed to the crankshaft S, while gear C is fixed. Gear B and the propeller are free to rotate. The crankshaft is turning at 80 rad/s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.

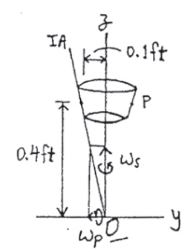


SOLUTION

Point P on gear B has a speed of

$$v_P = 80(0.4) = 32 \text{ ft/s}$$

The IA is located along the points of contant of B and C



$$\frac{\omega_P}{0.1} = \frac{\omega_s}{0.4}$$

$$\omega_s = 4\omega_P$$

$$\omega = -\omega_P\,\mathbf{j} + \omega_s\,\mathbf{k}$$

$$= -\omega_P\mathbf{j} + 4\omega_P\mathbf{k}$$

$$\mathbf{r}_{P/O} = 0.1\mathbf{j} = 0.4\mathbf{k}$$

$$\mathbf{v}_P = -32\mathbf{i}$$

$$\mathbf{v}_P = \omega \times \mathbf{r}_{P/O}$$

$$-32\mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\omega_P & 4\omega_P \\ 0 & 0.1 & 0.4 \end{vmatrix}$$

$$-32\mathbf{i} = -0.8\,\omega_P\mathbf{i}$$

$$\omega_P = 40 \text{ rad/s}$$

$$\omega_P = \{-40\mathbf{j}\} \text{ rad/s}$$ **Ans.**

$$\omega_s = 4(40)\,\mathbf{k} = \{160\mathbf{k}\} \text{ rad/s}$$

Thus,

$$\omega = \omega_P + \omega_s$$

Let the x,y,z axes have an angular velocity of $\Omega \times \omega_P$, then

$$\alpha = \dot{\omega} = \dot{\omega}_P + \dot{\omega}_s = \mathbf{0} + \omega_P \times (\omega_s + \omega_P)$$

$$\alpha = (-40\mathbf{j}) \times (160\mathbf{k} - 40\mathbf{j})$$

$$\alpha = \{-6400\mathbf{i}\} \text{ rad/s}^2$$ **Ans.**

Ans:
$\boldsymbol{\omega}_P = \{-40\mathbf{j}\}$ rad/s
$\boldsymbol{\alpha}_B = \{-6400\mathbf{i}\}$ rad/s^2

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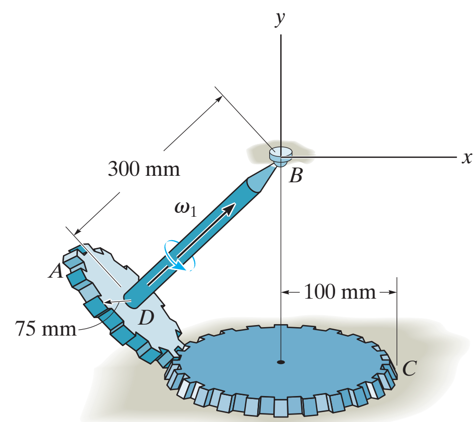
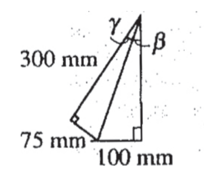
20–19.

Shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are *spinning* with a constant angular velocity $\omega_1 = 8$ rad/s, determine the angular velocity and angular acceleration of gear A.



SOLUTION

$$\gamma = \tan^{-1}\frac{75}{300} = 14.04^\circ \qquad \beta = \sin^{-1}\frac{100}{\sqrt{300^2 + 75^2}} = 18.87^\circ$$



The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

$$\frac{\omega}{\sin 147.09^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega = 13.44 \text{ rad/s}$$

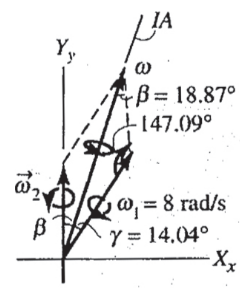
$$\omega = 13.44 \sin 18.87^\circ\,\mathbf{i} + 13.44 \cos 18.87^\circ\,\mathbf{j}$$

$$= \{4.35\mathbf{i} + 12.7\mathbf{j}\} \text{ rad/s}$$ **Ans.**

$$\frac{\omega_2}{\sin 14.04^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega_2 = 6.00 \text{ rad/s}$$

$$\omega_2 = \{6\mathbf{j}\} \text{ rad/s}$$

$$\omega_1 = 8 \sin 32.91^\circ\,\mathbf{i} + 8 \cos 32.91^\circ\,\mathbf{j} = \{4.3466\mathbf{i} + 6.7162\mathbf{j}\} \text{ rad/s}$$



For ω_1, $\Omega = \omega_2 = \{6\mathbf{j}\}$ rad/s

$$(\omega_1)_{xyz} = (\omega_1)_{xyz} + \Omega \times \omega_1$$

$$= \mathbf{0} + (6\mathbf{j}) \times (4.3466\mathbf{i} + 6.7162\mathbf{j})$$

$$= \{-26.08\mathbf{k}\} \text{ rad/s}^2$$

For ω_2, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (-26.08\mathbf{k}) = \{-26.1\mathbf{k}\} \text{ rad/s}^2$$ **Ans.**

Ans:
$\boldsymbol{\omega} = \{4.35\mathbf{i} + 12.7\mathbf{j}\}$ rad/s
$\boldsymbol{\alpha} = \{-26.1\mathbf{k}\}$ rad/s^2

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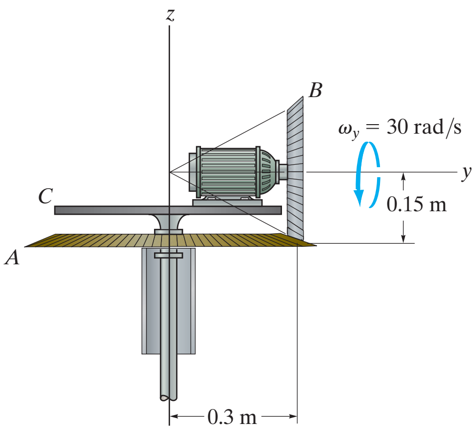
***20–20.**

Gear B is driven by a motor mounted on turntable C. If gear A is held fixed, and the motor shaft rotates with a constant angular velocity of $\omega_y = 30$ rad/s, determine the angular velocity and angular acceleration of gear B.



SOLUTION

The angular velocity ω of gear B is directed along the instantaneous axis of zero velocity, which is along the line where gears A and B mesh since gear A is held fixed. From Fig. a, the vector addition gives

$$\omega = \omega_y + \omega_z$$

$$\frac{2}{\sqrt{5}}\omega\mathbf{j} - \frac{1}{\sqrt{5}}\omega\mathbf{k} = 30\mathbf{j} - \omega_z\mathbf{k}$$

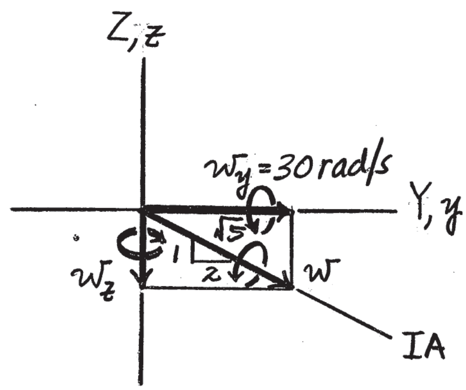
Equating the **j** and **k** components gives

$$\frac{2}{\sqrt{5}}\omega = 30 \qquad \omega = 15\sqrt{5}\text{ rad/s}$$

$$-\frac{1}{\sqrt{5}}\left(15\sqrt{5}\right) = -\omega_z \qquad \omega_z = 15\text{ rad/s}$$

Thus,

$$\omega = [30\mathbf{j} - 15\mathbf{k}]\text{ rad/s}$$ **Ans.**



(a)

Here, we will set the XYZ fixed reference frame to coincide with the xyz rotating frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_z = [-15\mathbf{k}]$ rad/s, then ω_y will always be directed along the y axis with respect to the xyz frame. Thus,

$$\dot{\omega}_y = \left(\dot{\omega}_y\right)_{xyz} + \omega_z \times \omega_y = 0 + (-15\mathbf{k}) \times (30\mathbf{j}) = [450\mathbf{i}]\text{ rad/s}^2$$

When $\Omega = \omega_z$, ω_z is always directed along the z axis. Therefore,

$$\dot{\omega}_z = \left(\dot{\omega}_z\right)_{xyz} + \omega_z \times \omega_z = 0 + 0 = 0$$

Thus,

$$\alpha = \dot{\omega}_y + \dot{\omega}_z = (450\mathbf{i}) + 0 = [450\mathbf{i}]\text{ rad/s}^2$$ **Ans.**

Ans:
$\omega = [30\mathbf{j} - 15\mathbf{k}]$ rad/s
$\alpha = [450\mathbf{i}]$ rad/s^2

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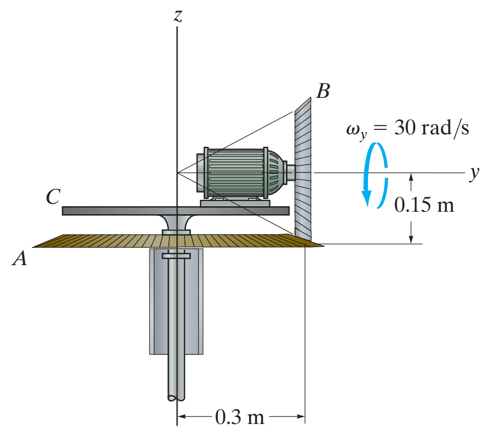
20–21.

Gear B is driven by a motor mounted on turntable C. If gear A and the motor shaft rotate with constant angular speeds of $\omega_A = \{10\mathbf{k}\}$ rad/s and $\omega_y = \{30\mathbf{j}\}$ rad/s, respectively, determine the angular velocity and angular acceleration of gear B.



SOLUTION

If the angular velocity of the turn-table is ω_z, then the angular velocity of gear B is

$$\omega = \omega_y + \omega_z = [30\mathbf{j} + \omega_z\mathbf{k}] \text{ rad/s}$$

Since gear A rotates about the fixed axis (z axis), the velocity of the contact point **P** between gears A and B is

$$\mathbf{v}_p = \omega_A \times r_A = (10\mathbf{k}) \times (0.3\mathbf{j}) = [-3\mathbf{i}] \text{ m/s}$$

Since gear B rotates about a fixed point O, the origin of the xyz frame, then $\mathbf{r}_{OP} = [0.3\mathbf{j} - 0.15\mathbf{k}]$ m.

$$\mathbf{v}_p = \omega \times \mathbf{r}_{OP}$$

$$-3\mathbf{i} = (30\mathbf{j} + \omega_z\mathbf{k}) \times (0.3\mathbf{j} - 0.15\mathbf{k})$$

$$-3\mathbf{i} = -(4.5 + 0.3\omega_z)\mathbf{i}$$

Thus,

$$-3 = -(4.5 + 0.3\omega_z)$$

$$\omega_z = -5 \text{ rad/s}$$

Then,

$$\omega = [30\mathbf{j} - 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

Here, we will set the XYZ fixed reference frame to conincide with the xyz rotating frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_z = [-5\mathbf{k}]$ rad/s, then ω_y will always be directed along the y axis with respect to the xyz frame. Thus,

$$\dot{\omega}_y = \left(\dot{\omega}_y\right)_{xyz} + \omega_z \times \omega_y = \mathbf{0} + (-5\mathbf{k}) \times (30\mathbf{j}) = [150\mathbf{i}] \text{ rad/s}^2$$

When $\Omega = \omega_z$, ω_z is always directed along the z axis. Therefore,

$$\dot{\omega}_z = \left(\dot{\omega}_z\right)_{xyz} + \omega_z \times \omega_z = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

Thus,

$$\alpha = \dot{\omega}_y + \dot{\omega}_z = (150\mathbf{i} + 0) = [150\mathbf{i}] \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega} = \{30\mathbf{j} - 5\mathbf{k}\}$ rad/s
$\boldsymbol{\alpha} = \{150\mathbf{i}\}$ rad/s^2

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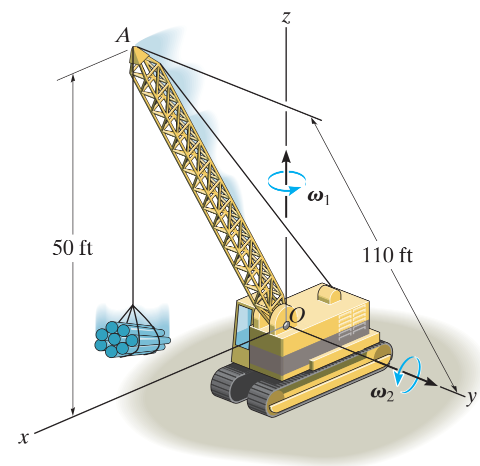
20–22.

The crane boom OA rotates about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity of $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



SOLUTION

$\omega = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\}$ rad/s

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$\dot{\omega} = (\dot{\omega})_{xyz} + \omega_1 \times \omega_2$

$\dot{\omega} = \mathbf{0} + 0.15\mathbf{k} \times 0.2\mathbf{j} = \{-0.03\mathbf{i}\}$ rad/s^2

$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\}$ ft

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\}$ ft/s **Ans.**

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}$ ft/s^2

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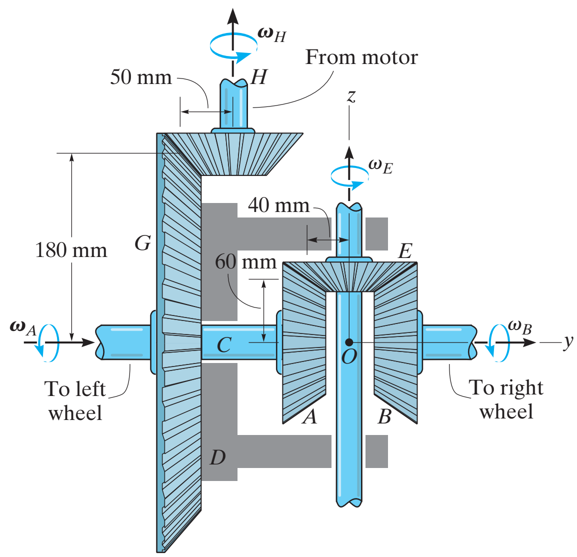
20–23.

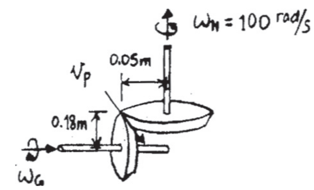
The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears A and B on their other ends. The differential case D is placed over the left axle but can rotate about C independent of the axle. The case supports a pinion gear E on a shaft, which meshes with gears A and B. Finally, a ring gear G is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning at $\omega_H = 100$ rad/s and the pinion gear E is spinning about its shaft at $\omega_E = 30$ rad/s, determine the angular velocity, ω_A and ω_B, of each axle.



SOLUTION

$$v_P = \omega_H r_H = 100(50) = 5000 \text{ mm/s}$$

$$\omega_G = \frac{5000}{180} = 27.78 \text{ rad/s}$$

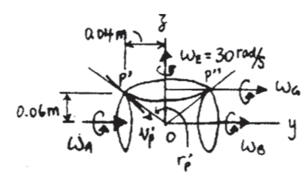


Point O is a fixed point of rotation for gears A, E, and B.

$$\Omega = \omega_G + \omega_E = \{27.78\mathbf{j} + 30\mathbf{k}\} \text{ rad/s}$$

$$\mathbf{v}_{P'} = \Omega \times \mathbf{r}_{P'} = (27.78\mathbf{j} + 30\mathbf{k}) \times (-40\mathbf{j} + 60\mathbf{k}) = \{2866.7\mathbf{i}\} \text{ mm/s}$$

$$\omega_A = \frac{2866.7}{60} = 47.8 \text{ rad/s} \qquad \textbf{Ans.}$$



$$\mathbf{v}_{P''} = \Omega \times \mathbf{r}_{P''} = (27.78\mathbf{j} + 30\mathbf{k}) \times (40\mathbf{j} + 60\mathbf{k}) = \{466.7\mathbf{i}\} \text{ mm/s}$$

$$\omega_B = \frac{466.7}{60} = 7.78 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_A = 47.8$ rad/s
$\omega_B = 7.78$ rad/s

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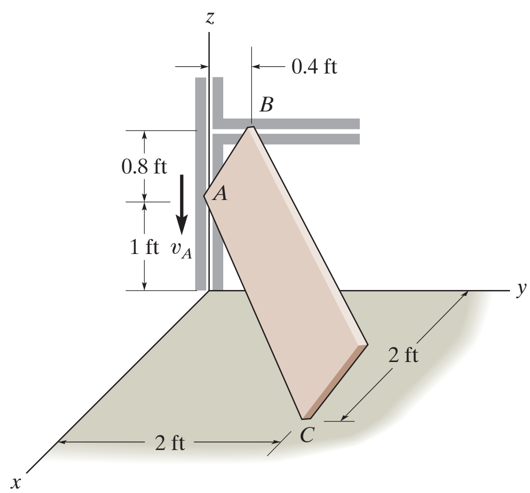
***20–24.**

The end C of the plate rests on the horizontal plane, while end points A and B are restricted to move along the grooved slots. If at the instant shown A is moving downward with a constant velocity of $v_A = 4$ ft/s, determine the angular velocity of the plate and the velocities of points B and C.



SOLUTION

Velocity equation:

$\mathbf{v}_A = \{-4\mathbf{k}\}$ ft/s $\quad \mathbf{v}_B = -v_B\mathbf{j} \quad \omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/A} = \{0.4\mathbf{j} + 0.8\mathbf{k}\}$ ft $\quad \mathbf{r}_{C/A} = \{2\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}\}$ ft

$\mathbf{v}_C = (v_C)_x\mathbf{i} + (v_C)_y\mathbf{j}$

$\mathbf{v}_B = v_A + \omega \times \mathbf{r}_{B/A}$

$$-v_B\mathbf{j} = (-4\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 0.4 & 0.8 \end{vmatrix}$$

Equating **i**, **j** and **k** components

$0.8\omega_y - 0.4\omega_z = 0$ **(1)**

$0.8\omega_x = v_B$ **(2)**

$0.4\omega_x - 4 = 0$ **(3)**

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{C/A}$$

$$(v_C)_x\mathbf{i} + (v_C)_y\mathbf{j} = (-4\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1 & 2 & -1 \end{vmatrix}$$

Equating **i**, **j** and **k** components

$-\omega_y - 2\omega_z = (v_c)_x$ **(4)**

$2\omega_z + \omega_x = (v_C)_y$ **(5)**

$2\omega_x - 2\omega_y - 4 = 0$ **(6)**

Solving Eqs. [1] to [6] yields:

$\omega_x = 10$ rad/s $\quad \omega_y = 8$ rad/s $\quad \omega_z = 16$ rad/s $\quad v_B = 8$ ft/s

$(v_C)_x = -40$ ft/s $\quad (v_C)_y = 42$ ft/s

Then $\mathbf{v}_B = \{-8\mathbf{j}\}$ ft/s $\quad \mathbf{v}_C = \{-40\mathbf{i} + 42\mathbf{j}\}$ ft/s **Ans.**

$\omega = \{10\mathbf{i} + 8\mathbf{j} + 16\mathbf{k}\}$ rad/s **Ans.**

Ans:
$\mathbf{v}_B = \{-8\mathbf{j}\}$ ft/s $\quad \mathbf{v}_C = \{-40\mathbf{i} + 42\mathbf{j}\}$ ft/s
$\omega = \{10\mathbf{i} + 8\mathbf{j} + 16\mathbf{k}\}$ rad/s

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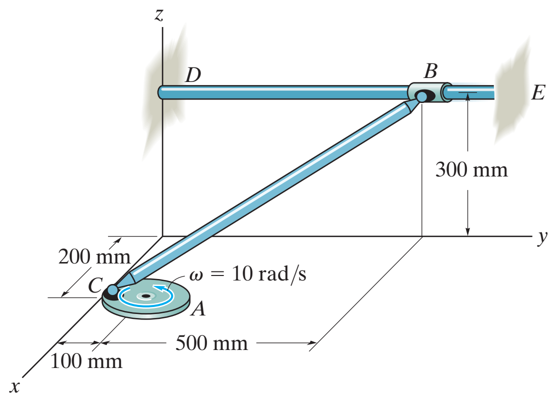
20–25.

Disk A rotates at a constant angular velocity of 10 rad/s. If rod BC is joined to the disk and a collar by ball-and-socket joints, determine the velocity of collar B at the instant shown. Also, what is the rod's angular velocity $\boldsymbol{\omega}_{BC}$ if it is directed perpendicular to the axis of the rod?



SOLUTION

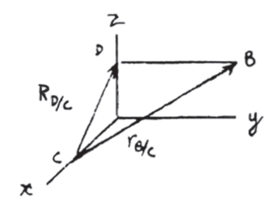
$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\qquad \mathbf{v}_B = -v_B\mathbf{j} \qquad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$r_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-\mathbf{v}_B = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components



$$1 - 0.3\omega_y - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_y = 0 \qquad \textbf{(3)}$$

Since ω_{BC} is perpendicular to the axis of the rod,

$$\omega_{BC} \cdot \mathbf{r}_{B/C} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}) \cdot (-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}) = 0$$

$$-0.2\omega_x + 0.6\omega_y + 0.3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.204$ rad/s $\qquad \omega_y = -0.612$ rad/s $\qquad \omega_z = 1.36$ rad/s $\qquad v_B = 0.333$ m/s

Then

$$\omega_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega}_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}$ rad/s
$\mathbf{v}_B = \{-0.333\mathbf{j}\}$ m/s

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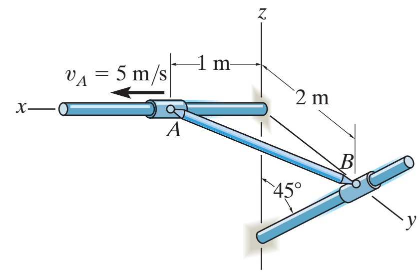
20–26.

Rod AB is attached to collars at its ends by using ball-and-socket joints. If collar A moves along the fixed rod at $v_A = 5$ m/s, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the axis of the rod.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{i}\}\ \text{m/s} \qquad \mathbf{v}_B = v_B \sin 45°\mathbf{j} + v_B \cos 45°\mathbf{k} = \frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k}$$

Also, $\mathbf{r}_{B/A} = (0 - 1)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{-1\mathbf{i} + 2\mathbf{j}\}$ m and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k} = 5\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j})\omega_z\mathbf{k} \times (-1\mathbf{i} + 2\mathbf{j})$$

$$\frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k} = (5 - 2\omega_z)\mathbf{i} - \omega_z\mathbf{j} + (2\omega_x + \omega_y)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = 5 - 2\,\omega_z \qquad \textbf{(1)}$$

$$\frac{1}{\sqrt{2}}v_B = -\omega_z \qquad \textbf{(2)}$$

$$\frac{1}{\sqrt{2}}v_B = 2\omega_x + \omega_y \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-1\mathbf{i} + 2\mathbf{j}) = 0$$

$$-\omega_x + 2\omega_y = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -1.00\ \text{rad/s} \quad \omega_y = -0.500\ \text{rad/s} \quad \omega_z = 2.50\ \text{rad/s} \quad v_B = -2.50\sqrt{2}\ \text{m/s}$$

Then

$$\boldsymbol{\omega}_{AB} = \{-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{j} + \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{k} = \{-2.50\mathbf{j} - 2.50\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

Note: v_B can be obtained by solving Eqs. 1 and 2 without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$\boldsymbol{\omega}_{AB} = \{-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}\}$ rad/s
$\mathbf{v}_B = \{-2.50\mathbf{j} - 2.50\mathbf{k}\}$ m/s

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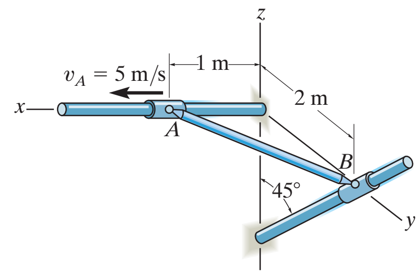
20–27.

Rod AB is attached to collars at its ends by using ball-and-socket joints. If collar A moves along the fixed rod with a velocity of $v_A = 5$ m/s and has an acceleration $a_A = 2$ m/s^2 at the instant shown, determine the angular acceleration of the rod and the acceleration of collar B at this instant. Assume that the rod's angular velocity and angular acceleration are directed perpendicular to the axis of the rod.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{i}\} \text{ m/s} \qquad \mathbf{v}_B = v_B \sin 45°\mathbf{j} + v_B \cos 45°\mathbf{k} = \frac{1}{\sqrt{2}} v_B\mathbf{j} + \frac{1}{\sqrt{2}} v_B\mathbf{k}$$

Also, $\mathbf{r}_{B/A} = (0 - 1)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{-1\mathbf{i} + 2\mathbf{j}\}$ m and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k} = 5\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j})\omega_z\mathbf{k} \times (-1\mathbf{i} + 2\mathbf{j})$$

$$\frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k} = (5 - 2\omega_z)\mathbf{i} - \omega_z\mathbf{j} + (2\omega_x + \omega_y)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = 5 - 2\,\omega_z \qquad \textbf{(1)}$$

$$\frac{1}{\sqrt{2}}v_B = -\omega_z \qquad \textbf{(2)}$$

$$\frac{1}{\sqrt{2}}v_B = 2\omega_x + \omega_y \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-1\mathbf{i} + 2\mathbf{j}) = 0$$

$$-\omega_x + 2\omega_y = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -1.00 \text{ rad/s} \quad \omega_y = -0.500 \text{ rad/s} \quad \omega_z = 2.50 \text{ rad/s} \quad v_B = -2.50\sqrt{2} \text{ m/s}$$

Then

$$\boldsymbol{\omega}_{AB} = \{-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{j} + \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{k} = \{-2.50\mathbf{j} - 2.50\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

Note: v_B can be obtained by solving Eqs. 1 and 2 without knowing the direction of $\boldsymbol{\omega}_{AB}$.

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20–27. Continued

The accelerations of collars A and B are

$$a_A = \{2\mathbf{i}\}\ \text{m/s}^2 \quad a_B = a_B \sin 45°\mathbf{j} + a_B \cos 45°\mathbf{k} = \frac{1}{\sqrt{2}}a_B\mathbf{j} + \frac{1}{\sqrt{2}}a_B\mathbf{k}$$

Also, $\alpha_{AB} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A})$$

$$\frac{1}{\sqrt{2}}a_B\mathbf{j} + \frac{1}{\sqrt{2}}a_B\mathbf{k} = 2\mathbf{i} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (-1\mathbf{i} + 2\mathbf{j})$$

$$+ (-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}) \times [(-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}) \times (-1\mathbf{i} + 2\mathbf{j})]$$

$$\frac{1}{\sqrt{2}}a_B\mathbf{j} + \frac{1}{\sqrt{2}}a_B\mathbf{k} = (9.5 - 2\alpha_z)\mathbf{i} + (-\alpha_z - 15)\mathbf{j} + (2\alpha_x + \alpha_y)\mathbf{k}$$

Equating **i**, **j** and **k** components

$$0 = 9.5 - 2\alpha_z \qquad \textbf{(5)}$$

$$\frac{1}{\sqrt{2}}a_B = -\alpha_z - 15 \qquad \textbf{(6)}$$

$$\frac{1}{\sqrt{2}}a_B = z\alpha_x + \alpha_y \qquad \textbf{(7)}$$

Assuming that $\boldsymbol{\alpha}_{AB}$ is directed perpendicular to the axis of rod AB, then

$$\boldsymbol{\alpha}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \cdot (-1\mathbf{i} + 2\mathbf{j}) = 0$$

$$-\alpha_x + 2x_y = 0 \qquad \textbf{(8)}$$

Solving Eqs. 5 to 8,

$$\alpha_x = -7.9\ \text{rad/s}^2 \quad \alpha_y = -3.95\ \text{rad/s}^2 \quad \alpha_z = 4.75\ \text{rad/s}^2 \quad a_B = -19.75\sqrt{2}\ \text{m/s}^2$$

Thus,

$$\boldsymbol{\alpha}_{AB} = \{-7.9\mathbf{i} - 3.95\mathbf{j} + 4.75\mathbf{k}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \frac{1}{\sqrt{2}}(-19.75\sqrt{2})\mathbf{j} + \frac{1}{\sqrt{2}}(-19.75\sqrt{2})\mathbf{j} = \{-19.75\mathbf{j} - 19.75\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\alpha}_{AB} = \{-7.9\mathbf{i} - 3.95\mathbf{j} + 4.75\mathbf{k}\}\ \text{rad/s}^2$
$\mathbf{a}_B = \{-19.75\mathbf{j} - 19.75\mathbf{k}\}\ \text{m/s}^2$

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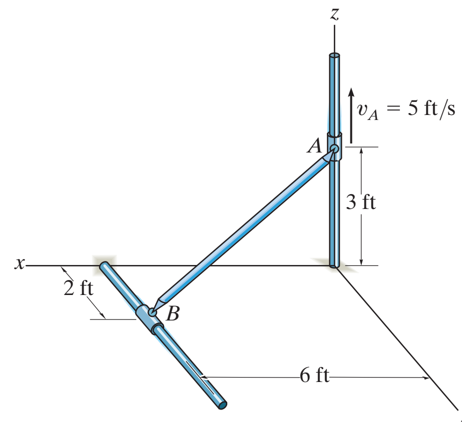
***20–28.**

If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the velocity of B at the instant shown if A is moving upward at a constant speed of $v_A = 5$ ft/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{k}\}\text{ ft/s} \quad \mathbf{v}_B = -v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (6 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ ft and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 5\mathbf{k} + \left(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\right) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$-v_B\mathbf{j} = \left(-3\omega_y - 2\omega_z\right)\mathbf{i} + \left(3\omega_x + 6\omega_z\right)\mathbf{j} + \left(2\omega_x - 6\omega_y + 5\right)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-v_B = 3\omega_x + 6\omega_z \qquad \textbf{(2)}$$

$$0 = 2\omega_x - 6\omega_y + 5 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\right) \cdot (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) = 0$$

$$6\omega_x + 2\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -\frac{65}{98}\text{ rad/s} = -0.6633\text{ rad/s} \quad \omega_y = \frac{30}{49}\text{ rad/s} = 0.6122\text{ rad/s}$$

$$\omega_z = -\frac{45}{49}\text{ rad/s} = -0.9183\text{ rad/s} \quad v_B = 7.50\text{ ft/s}$$

Thus,

$$\boldsymbol{\omega}_{AB} = \{-0.663\mathbf{i} + 0.612\mathbf{j} - 0.918\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-7.50\mathbf{j}\}\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega}_{AB} = \{-0.663\mathbf{i} + 0.612\mathbf{j} - 0.918\mathbf{k}\}$ rad/s
$\mathbf{v}_B = \{-7.50\mathbf{j}\}$ ft/s

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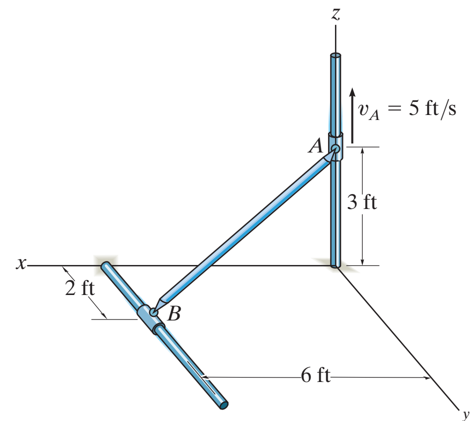
20–29.

If the collar at A in Prob. 20–28 is moving upward with an acceleration of $\mathbf{a}_A = \{-2\mathbf{k}\}\ \text{ft/s}^2$, at the instant its speed is $v_A = 5\ \text{ft/s}$, determine the acceleration of the collar at B at this instant.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{k}\}\ \text{ft/s} \qquad \mathbf{v}_B = -v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (6-0)\mathbf{i} + (2-0)\mathbf{j} + (0-3)\mathbf{k} = \{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ ft and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 5\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$-v_B\mathbf{j} = (-3\omega_y - 2\omega_z)\mathbf{i} + (3\omega_x + 6\omega_z)\mathbf{j} + (2\omega_x - 6\omega_y + 5)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-v_B = 3\omega_x + 6\omega_z \qquad \textbf{(2)}$$

$$0 = 2\omega_x - 6\omega_y + 5 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) = 0$$

$$6\omega_x + 2\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -\frac{65}{98}\ \text{rad/s} = -0.6633\ \text{rad/s} \quad \omega_y = \frac{30}{49}\text{rad/s} = 0.6122\ \text{rad/s}$$

$$\omega_z = -\frac{45}{49}\ \text{rad/s} = -0.9183\ \text{rad/s} \quad v_B = 7.50\ \text{ft/s}$$

Thus,

$$\boldsymbol{\omega}_{AB} = \{-0.663\mathbf{i} + 0.612\mathbf{j} - 0.918\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-7.50\mathbf{j}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

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20–29. Continued

The accelerations of collars A and B are

$$\mathbf{a}_A = \{-2\mathbf{k}\} \text{ ft/s}^2 \quad \mathbf{a}_B = a_B\mathbf{j}$$

Also, $\alpha_{AB} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A})$$

$$a_B\mathbf{j} = -2\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + (-0.6633\mathbf{i} + 0.6122\mathbf{j} - 0.9183\mathbf{k}) \times [(-0.6633\mathbf{i} + 0.6122\mathbf{j} - 0.9183\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})]$$

$$a_B\mathbf{j} = (-3\alpha_y - 2\alpha_z - 9.9490)\mathbf{i} + (3\alpha_x + 6\alpha_z - 3.3163)\mathbf{j} + (2\alpha_x - 6\alpha_y + 2.9745)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\alpha_y - 2\alpha_z - 9.9490 \qquad \textbf{(5)}$$

$$a_B = 3\alpha_x + 6\alpha_z - 3.3163 \qquad \textbf{(6)}$$

$$0 = 2\alpha_x - 6\alpha_y + 2.9745 \qquad \textbf{(7)}$$

Eliminate α_y from Eqs. 5 and 7

$$2\alpha_x + 4\alpha_z = -22.8724 \qquad \textbf{(8)}$$

Multiply Eq. 6 by $\frac{2}{3}$ and rearrange,

$$2\alpha_x + 4\alpha_z = \frac{2}{3}a_B + 2.2109 \qquad \textbf{(9)}$$

Equating Eqs. (8) and (9)

$$-22.8724 = \frac{2}{3}a_B + 2.2109$$

$$a_B = -37.625 \text{ ft/s}^2$$

Thus,

$$a_B = \{-37.6\mathbf{j}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Note: There is no need to know the direction of $\boldsymbol{\alpha}_{AB}$ to determine $\mathbf{a}_B$.

Ans:
$\mathbf{a}_B = \{-37.6\mathbf{j}\} \text{ ft/s}^2$

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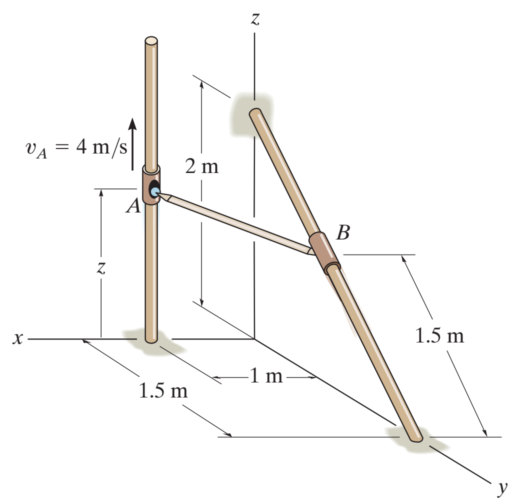
20–30.

Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed $v_A = 4$ m/s, determine the speed of collar B at the instant $z = 2$ m. Assume the angular velocity of the rod is directed perpendicular to the rod.



SOLUTION

$$v_B = v_A + \omega \times r_{B/A}$$

The velocities of collars A and B are

$$\mathbf{v}_A = \{4\mathbf{k}\}\ \text{m/s} \qquad \mathbf{v}_B = -v_B\left(\frac{3}{5}\right)\mathbf{j} + v_B\left(\frac{4}{5}\right)\mathbf{k} = -\frac{3}{5}v_B\mathbf{j} + \frac{4}{5}v_B\mathbf{k}$$

Also, the coordinates of points A and B are $A(1, 0, 2)$ m and $B\left\{0, \left[1.5 - 1.5\left(\frac{3}{5}\right)\right], 1.5\left(\frac{4}{5}\right)\right] = B(0, 0.6, 1.2)$ m. Thus, $\mathbf{r}_{B/A} = (0 - 1)\mathbf{i} + (0.6 - 0)\mathbf{j} + (1.2 - 2)\mathbf{k}$ $= \{-1\mathbf{i} + 0.6\mathbf{j} - 0.8\mathbf{k}\}$ m. Also $\omega_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$-\frac{3}{5}v_B\mathbf{j} + \frac{4}{5}v_B\mathbf{k} = 4\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (-1\mathbf{i} + 0.6\mathbf{j} - 0.8\mathbf{k})$$

$$-\frac{3}{5}v_B\mathbf{j} + \frac{4}{5}v_B\mathbf{k} = (-0.8\omega_y - 0.6\omega_z)\mathbf{i} + (0.8\omega_x - \omega_z)\mathbf{j} + (0.6\omega_x + \omega_y + 4)\mathbf{k}$$

Equating **i**, **j** and **k** components

$$0 = -0.8\omega_y - 0.6\omega_z \qquad \textbf{(1)}$$

$$-\frac{3}{5}v_B = 0.8\omega_x - \omega_z \qquad \textbf{(2)}$$

$$\frac{4}{5}v_B = 0.6\omega_x + \omega_y + 4 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of the rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-1\mathbf{i} + 0.6\mathbf{j} - 0.8\mathbf{k}) = 0$$

$$-\omega_x + 0.6\omega_y - 0.8\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4),

$$\omega_x = -1.20\ \text{rad/s} \quad \omega_y = -0.720\ \text{rad/s} \quad \omega_z = 0.960\ \text{rad/s}$$

$$v_B = 3.20\ \text{m/s}$$

Then $v_B = -\frac{3}{5}(3.20)\mathbf{j} - \frac{4}{5}(3.20)\mathbf{k} = \{-1.92\mathbf{j} + 2.56\mathbf{k}\}$ m/s **Ans.**

Note: v_B can also be obtained by Solving Eqs. (1) to (3) without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$\mathbf{v}_B = \{-1.92\mathbf{j} + 2.56\mathbf{k}\}$ m/s

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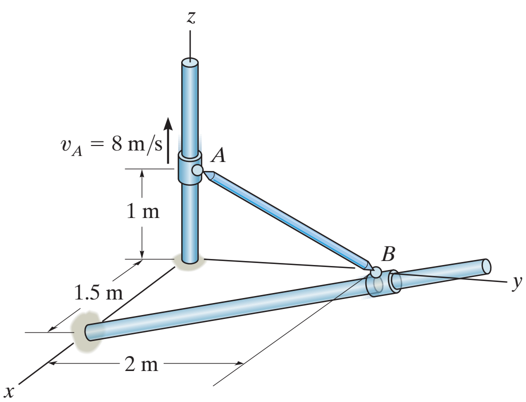
20–31.

The rod is attached to smooth collars A and B at its ends using ball-and-socket joints. Determine the speed of B at the instant shown if A is moving at $v_A = 8$ m/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.



SOLUTION

$\mathbf{v}_B = v_A + \mathbf{r}_{B/A}$

The velocities of collars A and B are

$$\mathbf{v}_A = \{8\mathbf{k}\}\ \text{m/s} \qquad \mathbf{v}_B = v_B\left(\frac{3}{5}\right)\mathbf{i} - v_B\left(\frac{4}{5}\right)\mathbf{j} = \frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (0 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 1)\mathbf{k} = \{2\mathbf{j} - 1\mathbf{k}\}$ m and $\boldsymbol{\omega}_{AB} + \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{3}{5}v_B\,\mathbf{i} - \frac{4}{5}v_B\,\mathbf{j} = 8\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$\frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j} = (-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x + 8)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$\frac{3}{5}v_B = -\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-\frac{4}{5}v_B = \omega_x \qquad \textbf{(2)}$$

$$0 = 2\omega_x + 8 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eq (1) to (4)

$$\omega_x = -4.00\ \text{rad/s} \qquad \omega_y = -0.600\ \text{rad/s} \qquad \omega_z = -1.20\ \text{rad/s}$$

$$v_B = 5.00\ \text{m/s} \qquad \textbf{Ans.}$$

Then,

$$\omega_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

Note. v_B can be obtained by solving Eqs (2) and (3) without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$v_B = 5.00$ m/s
$\boldsymbol{\omega}_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}$ rad/s

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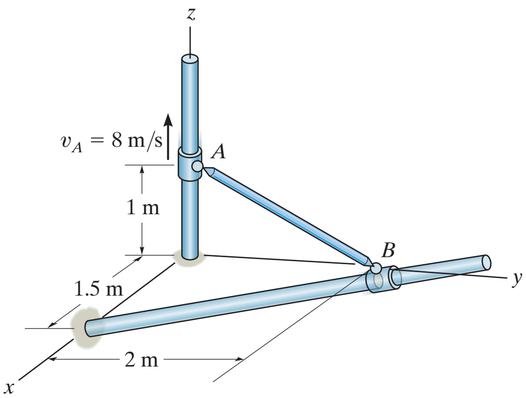
***20–32.**

If the collar A in Prob. 20–31 has a deceleration of $\mathbf{a}_A = \{-5\mathbf{k}\}\ \text{m/s}^2$, at the instant shown, determine the acceleration of collar B at this instant.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{r}_{B/A}$

The velocities of collars A and B are

$$\mathbf{v}_A = \{8\mathbf{k}\}\ \text{m/s} \qquad \mathbf{v}_B = v_B\left(\frac{3}{5}\right)\mathbf{i} - v_B\left(\frac{4}{5}\right)\mathbf{j} = \frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (0-0)\mathbf{i} + (2-0)\mathbf{j} + (0-1)\mathbf{k} = \{2\mathbf{j} - 1\mathbf{k}\}$ m and $\boldsymbol{\omega}_{AB} + \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j} = 8\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$\frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j} = (-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x + 8)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$\frac{3}{5}v_B = -\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-\frac{4}{5}v_B = \omega_x \qquad \textbf{(2)}$$

$$0 = 2\omega_x + 8 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eq (1) to (4)

$$\omega_x = -4.00\ \text{rad/s} \qquad \omega_y = -0.600\ \text{rad/s} \qquad \omega_z = -1.20\ \text{rad/s}$$

$$v_B = 5.00\ \text{m/s} \qquad \textbf{Ans.}$$

Then,

$$\omega_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

Note. v_B can be obtained by solving Eqs (2) and (3) without knowing the direction of $\boldsymbol{\omega}_{AB}$.

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***20–32. Continued**

The accelerations of collars A and B are

$$\mathbf{a}_A = \{-5\mathbf{k}\}\ \text{m/s}^2 \qquad \mathbf{a}_B = -a_B\left(\frac{3}{5}\right)\mathbf{i} + a_B\left(\frac{4}{5}\right)\mathbf{j} = -\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{j}$$

Also, $\alpha_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B})$$

$$-\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{j} = -5\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$
$$+ (-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}) \times [(-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})]$$

$$-\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{j} = (-\alpha_y - 2\alpha_z)\mathbf{i} + (\alpha_x - 35.6)\mathbf{j} + (2\alpha_x + 12.8)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$-\frac{3}{5}a_B = -\alpha_y + 2\alpha_z \qquad \textbf{(5)}$$

$$\frac{4}{5}a_B = \alpha_x - 35.6 \qquad \textbf{(6)}$$

$$0 = 2\alpha x + 12.8 \qquad \textbf{(7)}$$

Solving Eqs (6) and (7),

$$\alpha_x = -6.40\ \text{rad/s}^2 \qquad a_B = -52.5\ \text{m/s}^2$$

Then

$$a_B = -\frac{3}{5}(-52.5)\mathbf{i} + \frac{4}{5}(-52.5)\mathbf{j}$$

$$= \{31.5\mathbf{i} - 42.0\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Note. It is not necessary to know the direction of $\boldsymbol{\alpha}_{AB}$, if only $\mathbf{a}_B$ needs to be determined.

Ans:
$v_B = 5.00$ m/s
$\omega_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}$ rad/s
$a_B = \{31.5\mathbf{i} - 42.0\mathbf{j}\}$ m/s^2

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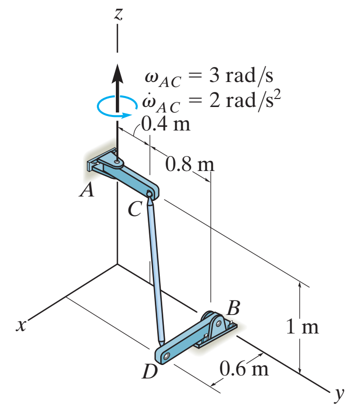
20–33.

Rod CD is attached to the rotating arms using ball-and-socket joints. If AC has the motion shown, determine the angular velocity of link BD at the instant shown.



SOLUTION

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/C}$

The velocities of points C and D are

$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{AC} = 3\mathbf{k} \times 0.4\mathbf{j} = \{-1.2\mathbf{i}\}\ \text{m/s}$

$\mathbf{v}_D = \boldsymbol{\omega}_{BD} \times \mathbf{r}_{BD} = \omega_{BD}\mathbf{j} \times 0.6\mathbf{i} = -0.6\omega_{BD}\mathbf{k}$

Also, $\mathbf{r}_{D/C} = (0.6 - 0)\mathbf{i} + (1.2 - 0.4)\mathbf{j} + (0.1)\mathbf{k} = \{0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}\}$ m and $\omega_{CD} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$

$-0.6\omega_{BD}\mathbf{k} = -1.2\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})$

$-0.6\omega_{BD}\mathbf{k} = (-\omega_y - 0.8\omega_z - 1.2)\mathbf{i} + (\omega_x + 0.6\omega_z)\mathbf{j} + (0.8\omega_x - 0.6\omega_y)\mathbf{k}.$

Equating **i**, **j** and **k** components,

$-\omega_y - 0.8\omega_z - 1.2 = 0$ **(1)**

$\omega_x + 0.6\omega_z = 0$ **(2)**

$0.8\omega_x - 0.6\omega_y = -0.6\omega_{BD}$ **(3)**

Assuming that $\boldsymbol{\omega}_{CD}$ is perpendicular to the axis of rod CD, then

$\boldsymbol{\omega}_{CD} \cdot \mathbf{r}_{D/C} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j}) + \omega_z\mathbf{k}) \cdot (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}) = 0$

$0.6\omega_x + 0.8\omega_y - \omega_z = 0$ **(4)**

Solving Eqs (1) to (4)

$\omega_x = 0.288\ \text{rad/s} \qquad \omega_y = -0.816\ \text{rad/s} \qquad \omega_z = -0.480\ \text{rad/s}$

$\omega_{BD} = -1.20\ \text{rad/s}$

Thus

$\omega_{BD} = \{-1.20\mathbf{j}\}\ \text{rad/s}$ **Ans.**

Note: $\boldsymbol{\omega}_{BD}$ can be obtained by solving Eqs 1 to 3 without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$\boldsymbol{\omega}_{BD} = \{-1.20\mathbf{j}\}\ \text{rad/s}$

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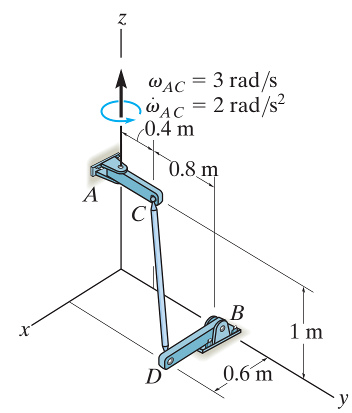
20–34.

Rod CD is attached to the rotating arms using ball-and-socket joints. If AC has the motion shown, determine the angular acceleration of link BD at this instant.



SOLUTION

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/C}$

The velocities of points C and D are

$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{AC} = 3\mathbf{k} \times 0.4\mathbf{j} = \{-1.2\mathbf{i}\}$ m/s

$\mathbf{v}_D = \boldsymbol{\omega}_{BD} \times \mathbf{r}_{BD} = \omega_{BD}\mathbf{j} \times 0.6\mathbf{i} = -0.6\omega_{BD}\mathbf{k}$

Also, $\mathbf{r}_{D/C} = (0.6 - 0)\mathbf{i} + (1.2 - 0.4)\mathbf{j} + (0.1)\mathbf{k} = \{0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}\}$ m and $\boldsymbol{\omega}_{CD} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$

$-0.6\omega_{BD}\mathbf{k} = -1.2\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})$

$-0.6\omega_{BD}\mathbf{k} = (-\omega_y - 0.8\omega_z - 1.2)\mathbf{i} + (\omega_x + 0.6\omega_z)\mathbf{j} + (0.8\omega_x - 0.6\omega_y)\mathbf{k}.$

Equating **i**, **j** and **k** components,

$-\omega_y - 0.8\omega_z - 1.2 = 0$ **(1)**

$\omega_x + 0.6\omega_z = 0$ **(2)**

$0.8\omega_x - 0.6\omega_y = -0.6\omega_{BD}$ **(3)**

Assuming that $\boldsymbol{\omega}_{CD}$ is perpendicular to the axis of rod CD, then

$\boldsymbol{\omega}_{CD} \cdot \mathbf{r}_{D/C} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j}) + \ \omega_z\mathbf{k}) \cdot (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}) = 0$

$0.6\omega_x + 0.8\omega_y - \omega_z = 0$ **(4)**

Solving Eqs (1) to (4)

$\omega_x = 0.288$ rad/s $\qquad \omega_y = -0.816$ rad/s $\qquad \omega_z = -0.480$ rad/s

$\omega_{BD} = -1.20$ rad/s

Thus

$\omega_{BD} = \{-1.20\mathbf{j}\}$ rad/s **Ans.**

Note: $\boldsymbol{\omega}_{BD}$ can be obtained by solving Eqs 1 to 3 without knowing the direction of $\boldsymbol{\omega}_{CD}$.

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20–34. Continued

The accelerations of points C and D are

$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{AC} - \omega_{AC}{}^2\mathbf{r}_{AC} = (2\mathbf{k} \times 0.4\mathbf{j}) - 3^2(0.4\mathbf{j}) = \{-0.8\mathbf{i}, -3.6\mathbf{j}\}\ \text{m/s}$

$\mathbf{a}_D = \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{BD} - \omega_{BD}{}^2\mathbf{r}_{BD} = (\alpha_{BD}\mathbf{j} \times 0.6\mathbf{i}) - 1.20^2(0.6\mathbf{i}) = -0.864\mathbf{i} - 0.6\alpha_{BD}\mathbf{k}$

Also, $\alpha_{CD} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$ and $\boldsymbol{\omega}_{CD} = \{0.288\mathbf{i} - 0.816\mathbf{j} - 0.480\mathbf{k}\}\ \text{rad/s}$

Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} + \boldsymbol{\omega}_{CD} \times (\boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C})$$

$$-0.864\mathbf{i} - 0.6\alpha_{BD}\mathbf{k} = (-0.8\mathbf{i} - 3.6\mathbf{j}) + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})$$
$$+ (0.288\mathbf{i} - 0.816\mathbf{j} - 0.480\mathbf{k}) \times [(0.288\mathbf{i} - 0.816\mathbf{j} - 0.480\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})]$$

$$-0.864\mathbf{i} - 0.6\alpha_{BD}\mathbf{k} = (-\alpha_y - 0.8\alpha_z - 1.38752)\mathbf{i} + (\alpha_x + 0.6\alpha_z - 4.38336)\mathbf{j}$$
$$+ (0.8\alpha_x - 0.6\alpha_y + 0.9792)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$-0.864 = -\alpha_y - 0.8\alpha_z - 1.38752 \qquad \textbf{(5)}$$

$$0 = \alpha_x + 0.6\alpha_z - 4.38336 \qquad \textbf{(6)}$$

$$-0.6\alpha_{BD} = 0.8\alpha_x - 0.6\alpha_y + 0.9792 \qquad \textbf{(7)}$$

Assuming that $\boldsymbol{\alpha}_{CD}$ is perpendicular to the axis of rod CD, then

$$\boldsymbol{\alpha}_{CD} \cdot \mathbf{r}_{D/C} = 0$$

$$(\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \cdot (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}) = 0$$

$$0.6\alpha_x + 0.8\alpha_y - \alpha_z = 0 \qquad \textbf{(8)}$$

Solving Eqs. 5 to 8,

$$\alpha_x = 3.72\ \text{rad/s}^2 \qquad \alpha_y = -1.408\ \text{rad/s}^2 \qquad \alpha_z = 1.1056\ \text{rad/s}^2$$
$$\alpha_{BD} = -8.00\ \text{rad/s}^2$$

Thus,

$$\alpha_{BD} = \{-8.00\mathbf{j}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Note: α_{BD} can be obtained by solving Eqs 5 to 7 without knowing the direction of $\boldsymbol{\alpha}_{CD}$.

Ans:
$\boldsymbol{\alpha}_{BD} = \{-8.00\mathbf{j}\}\ \text{rad/s}^2$

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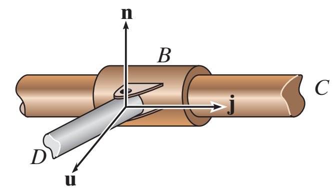
20–35.

Solve Prob. 20–28 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both along the bar (**j** direction) and along the axis of the pin (**n** direction). Since there is no rotational component in the **u** direction, i.e., perpendicular to **n** and **j** where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector **n** is in the same direction as $\mathbf{r}_{D/B} \times \mathbf{r}_{C/B}$.



SOLUTION

The velocities of collars A and B are

$\mathbf{v}_A = \{5\,\mathrm{k}\}$ ft/s $\qquad \mathbf{v}_B = -v_B\mathbf{j}$

Also, $\mathbf{r}_{B/A} = (6-0)\mathbf{i} + (2-0)\mathbf{j} + (0-3)\mathbf{k} = \{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ ft and $\omega_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 5\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$-v_B\mathbf{j} = (-3\omega_y - 2\omega_z)\mathbf{i} + (3\omega_x + 6\omega_z)\mathbf{j} + (2\omega_x - 6\omega_y + 5)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-v_B = 3\omega_x + 6\omega_z \qquad \textbf{(2)}$$

$$0 = 2\omega_x - 6\omega_y + 5 \qquad \textbf{(3)}$$

Here,

$$\mathbf{r}_{B/A} \times \mathbf{j} = (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \times \mathbf{j} = 3\mathbf{i} + 6\mathbf{k}$$

Then

$$\mathbf{n} = \frac{\mathbf{r}_{B/A} \times \mathbf{j}}{|\mathbf{r}_{B/A} \times \mathbf{j}|} = \frac{3\mathbf{i} + 6\mathbf{k}}{\sqrt{3^2 + 6^2}} = \frac{3}{\sqrt{45}}\mathbf{i} + \frac{6}{\sqrt{45}}\mathbf{k}$$

Thus

$$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times \left(\frac{3}{\sqrt{45}}\mathbf{i} + \frac{6}{\sqrt{45}}\mathbf{k}\right) = \frac{6}{\sqrt{45}}\mathbf{i} - \frac{3}{\sqrt{45}}\mathbf{k}$$

It is required that

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{u} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot \left(\frac{6}{\sqrt{45}}\mathbf{i} - \frac{3}{\sqrt{45}}\mathbf{k}\right) = 0$$

$$\frac{6}{\sqrt{45}}\omega_x - \frac{3}{\sqrt{45}}\omega_z = 0$$

$$2\omega_x - \omega_z = 0 \qquad \textbf{(4)}$$

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20–35. Continued

Solving Eqs (1) to (4)

$$\omega_x = -0.500 \text{ rad/s} \qquad \omega_y = 0.6667 \text{ rad/s} \qquad \omega_z = -1.00 \text{ rad/s}$$

$$v_B = 7.50 \text{ ft/s}$$

Thus,

$\boldsymbol{\omega}_{AB} = \{-0.500\mathbf{i} + 0.667\mathbf{j} - 1.00\mathbf{k}\} \text{ rad/s}$ **Ans.**

$v_B = \{-7.50\mathbf{j}\} \text{ ft/s}$ **Ans.**

Ans:
$\boldsymbol{\omega}_{AB} = \{-0.500\mathbf{i} + 0.667\mathbf{j} - 1.00\mathbf{k}\} \text{ rad/s}$
$\mathbf{v}_B = \{-7.50\mathbf{j}\} \text{ ft/s}$

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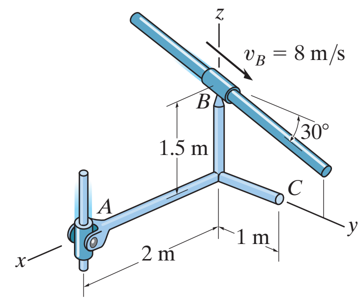
***20–36.**

Member ABC is pin connected at A and has a ball-and-socket joint at B. If the collar at B is moving along the inclined rod at $v_B = 8$ m/s, determine the velocity of point C at the instant shown. *Hint*: See Prob. 20–35.



SOLUTION

Velocities of collars A and B are

$$\mathbf{v}_A = v_A\,\mathbf{k} \qquad \mathbf{v}_B = 8\cos 30°\,\mathbf{j} - 8\sin 30°\,\mathbf{k} = \{4\sqrt{3}\,\mathbf{j} - 4\,\mathbf{k}\}\ \text{m/s}$$

Also, $\mathbf{r}_{A/B} = \{2\,\mathbf{i} - 1.5\,\mathbf{k}\}$ m and $\omega_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{k} = \left(4\sqrt{3}\mathbf{j} - 4\mathbf{k}\right) + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{i} - 1.5\mathbf{k})$$

$$v_A\mathbf{k} = -1.5\omega_y\mathbf{i} + \left(1.5\omega_x + 2\omega_z + 4\sqrt{3}\right)\mathbf{j} + (-2\omega_y - 4)\mathbf{k}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components,

$$0 = -1.5\,\omega_y \qquad \omega_y = 0$$

$$0 = 1.5\,\omega_x + 2\omega_z + 4\sqrt{3} \qquad \textbf{(1)}$$

$$v_A = -2(0) - 4 \qquad v_A = -4\ \text{m/s}$$

Here $\mathbf{n} = \mathbf{j}$. Then $\mathbf{u} = \mathbf{k} \times \mathbf{n} = \mathbf{k} \times \mathbf{j} = -\mathbf{i}$. It is required that

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{u} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-\mathbf{i}) = 0$$

$$-\omega_x = 0 \qquad \omega_x = 0$$

Substitute this result into Eq (1),

$$0 = 1.5(0) + 2\,\omega_z + 4\sqrt{3}$$

$$\omega_z = -2\sqrt{3}\ \text{rad/s}$$

Thus,

$$\omega_{AB} = \{-2\sqrt{3}\mathbf{k}\}\ \text{rad/s}$$

Here, $\mathbf{r}_{C/B} = \{1\mathbf{j} - 1.5\mathbf{k}\}$. Using the result of $\boldsymbol{\omega}_{AB}$,

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{C/B}$$

$$\mathbf{v}_C = (4\sqrt{3}\mathbf{j} - 4\mathbf{k}) + (-2\sqrt{3}\mathbf{k}) \times (1\mathbf{j} - 1.5\mathbf{k})$$

$$= \{2\sqrt{3}\mathbf{i} + 4\sqrt{3}\mathbf{j} - 4\mathbf{k}\}\ \text{m/s}$$

$$= \{3.46\mathbf{i} + 6.93\mathbf{j} - 4\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{3.46\mathbf{i} + 6.93\mathbf{j} - 4\mathbf{k}\}$ m/s

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20–37.

Solve Example 20.5 such that the x, y, z axes move with curvilinear translation, $\Omega = \mathbf{0}$ in which case the collar appears to have both an angular velocity $\mathbf{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$ and radial motion.

SOLUTION

Relative to XYZ, let xyz have

$$\Omega = 0 \qquad \dot{\Omega} = 0$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \{2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\}\text{ m/s}^2$$

Relative to xyz, let $x'\ y'\ z'$ be coincident with xyz and be fixed to BD. Then

$$\Omega_{xyz} = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\}\text{ rad/s} \qquad \dot{\omega}_{xyz} = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\}\text{ rad/s}^2$$

$$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\}\text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+\left[(\dot{\omega}_1 + \dot{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\}\text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + 0 + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}\text{ m/s}$
$\mathbf{a}_C = \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2$

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20–38.

Solve Example 20.5 by fixing x, y, z axes to rod BD so that $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$. In this case the collar appears only to move radially outward along BD; hence $\boldsymbol{\Omega}_{xyz} = \mathbf{0}$.

SOLUTION

Relative to XYZ, let $x'\ y'\ z'$ be concident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}\ \text{rad/s}$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \left[\left(\dot{\omega}_1\right)_{x'y'z'} + \omega_1 \times \omega_1\right] + \left[\left(\omega_2\right)_{x'y'z'} + \omega_1 \times \omega_2\right]$$

$$= (1.5\mathbf{i} + \mathbf{0}) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\ \text{m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{a}_B = \dot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \left(\dot{\mathbf{r}}_\mathbf{B}\right)_{x'y'z'}\right] + \dot{\omega}_1 \times r_B + \omega_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\}\ \text{m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\Omega_{x'y'z'} = \mathbf{0}; \qquad \dot{\Omega}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\}\ \text{m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\}\ \text{m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

$$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}$ m/s^2

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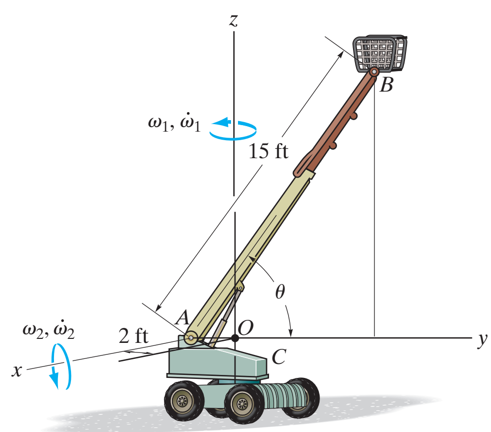
20–39.

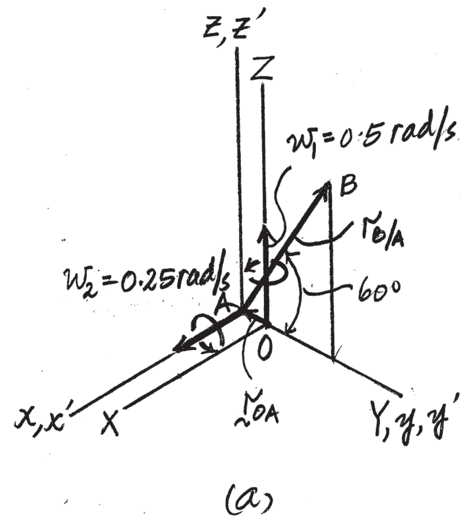
At the instant $\theta = 60°$, the telescopic boom AB of the construction lift is rotating with a constant angular velocity about the z axis of $\omega_1 = 0.5$ rad/s and about the pin at A with a constant angular speed of $\omega_2 = 0.25$ rad/s. Simultaneously, the boom is extending with a velocity of 1.5 ft/s, and it has an acceleration of 0.5 ft/s^2, both measured relative to the construction lift. Determine the velocity and acceleration of point B located at the end of the boom at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = \{0.5\mathbf{k}\}\text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = 0$$



Since point A rotates about a fixed axis (Z axis), its motion can determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (0.5\mathbf{k}) \times (-2\mathbf{j}) = \{1\mathbf{i}\}\text{ ft/s}$$

and

$$\begin{aligned} a_A &= \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA}) \\ &= 0 + (0.5\mathbf{k}) \times (0.5\mathbf{k}) \times (-2\mathbf{j}) \\ &= \{0.5\mathbf{j}\}\text{ ft/s}^2 \end{aligned}$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = \{0.25\mathbf{i}\}$ rad/s, the direction of $\mathbf{r}_{B/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{B/A}$,

$$\begin{aligned} (\mathbf{v}_{B/A})_{xyz} &= (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times \mathbf{r}_{B/A}\right] \\ &= (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k}) + 0.25\mathbf{i} \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) \\ &= \{-2.4976\mathbf{j} + 3.1740\mathbf{k}\}\text{ ft/s} \end{aligned}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = 0$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$\begin{aligned} (\mathbf{a}_{B/A})_{xyz} &= (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{B/A} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz} \\ &= \left[(0.5\cos 60°\mathbf{j} + 0.5\sin 60°\mathbf{k}) + 0.25\mathbf{i} \times (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k})\right] + 0.25\mathbf{i} \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) \\ &= \{-0.8683\mathbf{j} - 0.003886\mathbf{k}\}\text{ ft/s}^2 \end{aligned}$$

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20–39. Continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (1\mathbf{i}) + (0.5\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) + (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= 0.5\mathbf{j} + 0 + 0.5\mathbf{k} \times \left[(0.5\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k})\right]$$

$$+ 2(0.5\mathbf{k}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) + (-0.8683\mathbf{j} - 0.003886\mathbf{k})$$

$$= \{2.50\mathbf{i} - 2.24\mathbf{j} - 0.00389\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_B = \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{2.50\mathbf{i} - 2.24\mathbf{j} - 0.00389\mathbf{k}\}$ ft/s^2

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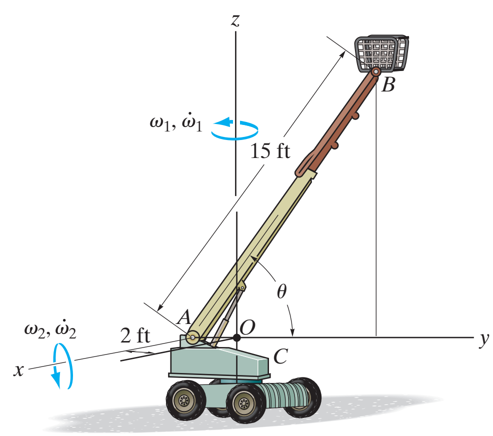
***20–40.**

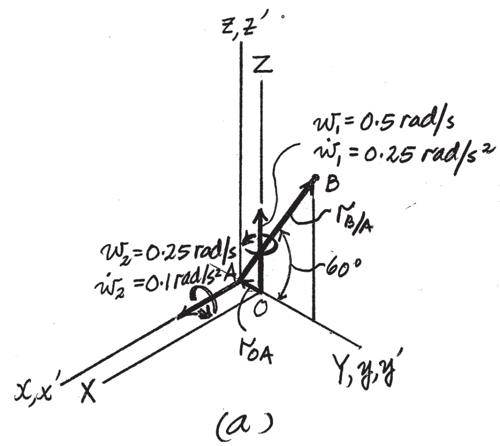
At the instant $\theta = 60°$, the construction lift is rotating about the z axis with an angular velocity of $\omega_1 = 0.5$ rad/s and an angular acceleration of $\dot{\omega}_1 = 0.25\ \text{rad/s}^2$ while the telescopic boom AB rotates about the pin at A with an angular velocity of $\omega_2 = 0.25$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.1\ \text{rad/s}^2$. Simultaneously, the boom is extending with a velocity of 1.5 ft/s, and it has an acceleration of 0.5 ft/s^2, both measured relative to the frame. Determine the velocity and acceleration of point B located at the end of the boom at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = \{0.5\mathbf{k}\}\ \text{rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \{0.25\mathbf{k}\}\ \text{rad/s}^2$$



Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (0.5\mathbf{k}) \times (-2\mathbf{j}) = \{1\mathbf{i}\}\ \text{ft/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.25\mathbf{k}) \times (-2\mathbf{j}) + (0.5\mathbf{k}) \times [0.5\mathbf{k} \times (-2\mathbf{j})]$$

$$= \{0.5\mathbf{i} + 0.5\mathbf{j}\}\ \text{ft/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = [0.25\mathbf{i}]$ rad/s, the direction of $\mathbf{r}_{B/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{B/A}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times \mathbf{r}_{B/A}\right]$$

$$= (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k}) + [0.25\mathbf{i} \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k})]$$

$$= \{-2.4976\mathbf{j} + 3.1740\mathbf{k}\}\ \text{ft/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = [0.1\mathbf{i}]\ \text{rad/s}^2$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A})_{xyz} = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{B/A} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= (0.5\cos 60°\mathbf{j} + 0.5\sin 60°\mathbf{k}) + (0.25\mathbf{i}) \times (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k})$$

$$+ (0.1\mathbf{i}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) + (0.25\mathbf{i}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.1673\mathbf{j} + 0.7461\mathbf{k}\}\ \text{ft/s}^2$$

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20–40. Continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= [1\mathbf{i}] + (0.5\mathbf{k}) \times (15 \cos 60°\mathbf{j} + 15 \sin 60°\mathbf{k}) + (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\} \text{ ft/s}$$ **Ans.**

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (0.5\mathbf{i} + 0.5\mathbf{j}) + (0.25\mathbf{k}) \times (15 \cos 60°\mathbf{j} + 15 \sin 60°\mathbf{k}) + (0.5\mathbf{k})$$

$$\times [(0.5\mathbf{k}) \times (15 \cos 60°\mathbf{j} + 15 \sin 60°\mathbf{k})] + 2\,(0.5\mathbf{k})$$

$$\times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) + (-2.1673\mathbf{j} + 0.7461\mathbf{k})$$

$$= \{1.12\mathbf{i} - 3.54\mathbf{j} + 0.746\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_B = \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}$ ft/s
$\mathbf{a}_B = \{1.12\mathbf{i} - 3.54\mathbf{j} + 0.746\mathbf{k}\}$ ft/s^2

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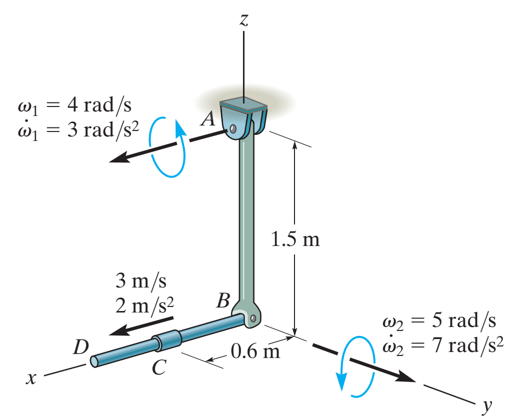
20–41.

At the instant shown, the arm AB is rotating about the fixed pin A with an angular velocity $\omega_1 = 4$ rad/s and angular acceleration $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, rod BD is rotating relative to rod AB with an angular velocity $\omega_2 = 5$ rad/s, which is increasing at $\dot{\omega}_2 = 7$ rad/s^2. Also, the collar C is moving along rod BD with a velocity of 3 m/s and an acceleration of 2 m/s^2, both measured relative to the rod. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

$\Omega = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega} = \{3\mathbf{i}\}$ rad/s^2

$\mathbf{r}_B = \{-1.5\mathbf{k}\}$ m

$$\mathbf{v}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B$$
$$= 0 + (4\mathbf{i}) \times (-1.5\mathbf{k})$$
$$= \{6\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$
$$= \mathbf{0} + \mathbf{0} + (3\mathbf{i}) \times (-1.5\mathbf{k}) + (4\mathbf{i}) \times (6\mathbf{j})$$
$$= \{4.5\mathbf{j} + 24\mathbf{k}\} \text{ m/s}^2$$

$\Omega_{C/B} = \{5\mathbf{j}\}$ rad/s

$\dot{\Omega}_{C/B} = \{7\mathbf{j}\}$ rad/s^2

$\mathbf{r}_{C/B} = \{0.6\mathbf{i}\}$ m

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times \mathbf{r}_{C/B}$$
$$= (3\mathbf{i}) + (5\mathbf{j}) \times (0.6\mathbf{i})$$
$$= \{3\mathbf{i} - 3\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right] + \dot{\Omega}_{C/B} \times \mathbf{r}_{C/B} + \Omega_{C/B} \times \dot{\mathbf{r}}_{C/B}$$
$$= (2\mathbf{i}) + (5\mathbf{j}) \times (3\mathbf{i}) + (7\mathbf{j}) \times (0.6\mathbf{i}) + (5\mathbf{j}) \times (3\mathbf{i} - 3\mathbf{k})$$
$$= \{-13\mathbf{i} - 34.2\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$
$$= (6\mathbf{j}) + (4\mathbf{i}) \times (0.6\mathbf{i}) + (3\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{v}_C = \{3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$
$$= (4.5\mathbf{j} + 24\mathbf{k}) + (3\mathbf{i}) \times (0.6\mathbf{i}) + (4\mathbf{i}) \times [(4\mathbf{i}) \times (0.6\mathbf{i})] + 2(4\mathbf{i}) \times (3\mathbf{i} - 3\mathbf{k}) + (-13\mathbf{i} - 34.2\mathbf{k})$$

$$\mathbf{a}_C = \{-13.0\mathbf{i} + 28.5\mathbf{j} - 10.2\mathbf{k}\} \text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-13.0\mathbf{i} + 28.5\mathbf{j} - 10.2\mathbf{k}\}$ m/s^2

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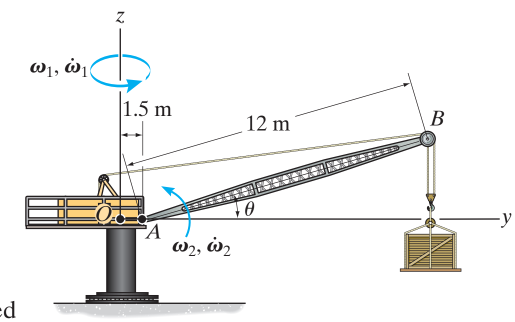
20–42.

At the instant $\theta = 30^\circ$, the frame of the crane and the boom AB rotate with a constant angular velocity of $\omega_1 = 1.5$ rad/s and $\omega_2 = 0.5$ rad/s, respectively. Determine the velocity and acceleration of point B at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}]\text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$

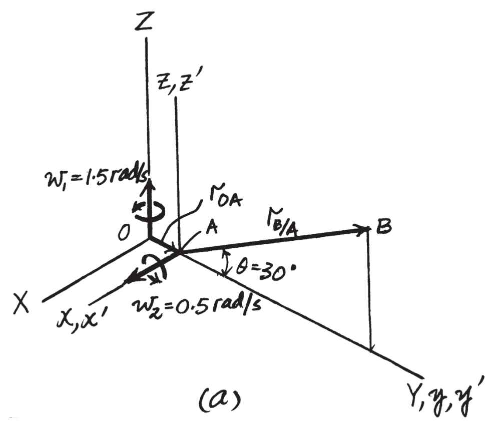
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}]\text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= \mathbf{0} + (1.5\mathbf{k}) \times [(1.5\mathbf{k}) \times (1.5\mathbf{j})]$$

$$= [-3.375\mathbf{j}]\text{ m/s}^2$$



In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12\cos 30^\circ\,\mathbf{j} + 12\sin 30^\circ\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}]\text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{A/B})_{xyz}$,

$$(\mathbf{a}_{A/B})_{xyz} = (\ddot{\mathbf{r}}_{A/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/B})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{A/B})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{xyz}$$

$$= [0 + 0] + 0 + (0.5\mathbf{i}) \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-2.598\mathbf{j} - 1.5\mathbf{k}]\text{ m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12\cos 30^\circ\,\mathbf{j} + 12\sin 30^\circ\,\mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}]\text{ m/s}$$ **Ans.**

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{AB})_{xyz} + (\mathbf{a}_{AB})_{xyz}$$

$$= (-3.375\mathbf{j}) + 0 + 1.5\mathbf{k} \times \left[(1.5\mathbf{k}) \times (12\cos 30^\circ\,\mathbf{j} + 12\sin 30^\circ\,\mathbf{k})\right] + 2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-2.598\mathbf{j} - 1.5\mathbf{k})$$

$$= [9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}]\text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_B = \{-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}\}$ m/s^2

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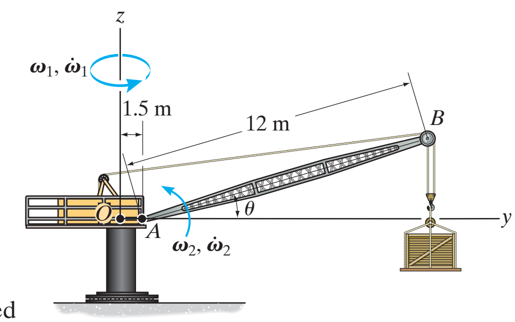
20–43.

At the instant $\theta = 30°$, the frame of the crane is rotating with an angular velocity of $\omega_1 = 1.5$ rad/s and angular acceleration of $\dot{\omega}_1 = 0.5$ rad/s^2, while the boom AB rotates with an angular velocity of $\omega_2 = 0.5$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.25$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = [0.5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.5\mathbf{k}) \times (1.5\mathbf{j}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (1.5\mathbf{j})\right]$$

$$= [-0.75\mathbf{i} - 3.375\mathbf{j}] \text{ m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \,\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [0.25\mathbf{i}]$ m/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A}) = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\Omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + (0.25\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \mathbf{k}) + 0.5\mathbf{i} \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-4.098\mathbf{j} + 1.098\mathbf{k}] \text{ m/s}^2$$

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20–43. Continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30^\circ\,\mathbf{j} + 12 \sin 30^\circ\mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

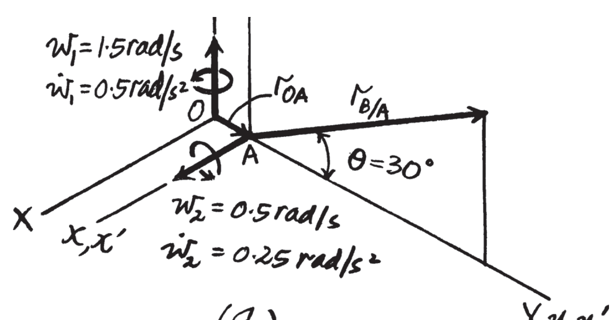
and

$$\mathbf{a}_B = \mathbf{a}_A = \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-0.75\mathbf{i} - 3.375\mathbf{j}) + 0.5\mathbf{k} \times (12 \cos 30^\circ\mathbf{j} + 12 \sin 30^\circ\,\mathbf{k}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (12 \cos 30^\circ\,\mathbf{j} + 12 \sin 30^\circ\,\mathbf{k})\right]$$

$$+2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-4.098\mathbf{j} + 1.098\mathbf{k})$$

$$= [3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$



(a)

Ans:
$\mathbf{v}_B = \{-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}\}$ m/s^2

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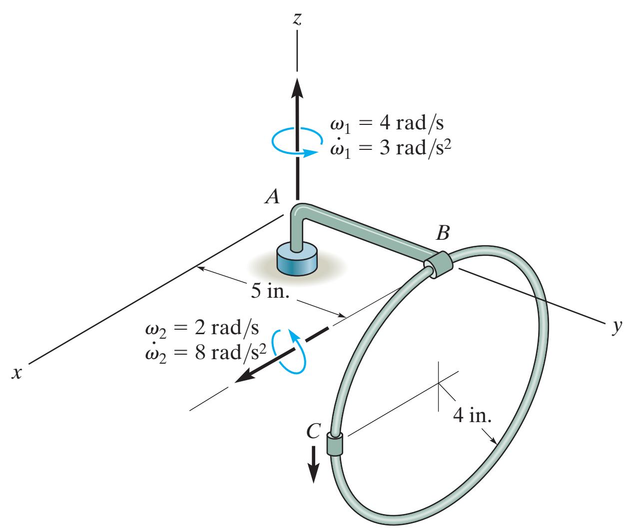
***20–44.**

At the instant shown, the rod AB is rotating about the z axis with an angular velocity $\omega_1 = 4$ rad/s and an angular acceleration $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, the circular rod has an angular motion relative to the rod as shown. If the collar C is moving down around the circular rod with a speed of 3 in./s, which is increasing at 8 in./s^2, both measured relative to the rod, determine the collar's velocity and acceleration at this instant.



SOLUTION

$$\Omega = \{4\mathbf{k}\} \text{ rad/s}$$

$$\dot{\Omega} = \{3\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_B = \{5\mathbf{j}\} \text{ in.}$$

$$\mathbf{v}_B = (4\mathbf{k}) \times (5\mathbf{j}) = \{-20\mathbf{i}\} \text{ in./s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= (4\mathbf{k}) \times (-20\mathbf{i}) + (3\mathbf{k}) \times (5\mathbf{j})$$

$$= \{-15\mathbf{i} - 80\mathbf{j}\} \text{ in./s}^2$$

$$\Omega_{C/B} = \{2\mathbf{i}\} \text{ rad/s}$$

$$\dot{\Omega}_{C/B} = \{8\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_{C/B} = \{4\mathbf{i} - 4\mathbf{k}\} \text{ in.}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times \mathbf{r}_{C/B}$$

$$= (-3\mathbf{k}) + (2\mathbf{i}) \times (4\mathbf{i} - 4\mathbf{k})$$

$$= \{8\mathbf{j} - 3\mathbf{k}\} \text{ in./s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right] + \dot{\Omega}_{C/B} \times \mathbf{r}_{C/B} + \Omega_{C/B} \times \dot{\mathbf{r}}_{C/B}$$

$$= \left(-\frac{3^2}{4}\mathbf{j} - 8\mathbf{k}\right) + (2\mathbf{i}) \times (-3\mathbf{k}) + (8\mathbf{i}) \times (4\mathbf{i} - 4\mathbf{k})$$
$$+ (2\mathbf{i}) \times (8\mathbf{j} - 3\mathbf{k})$$

$$= \{-2.25\mathbf{i} + 44\mathbf{j} + 8\mathbf{k}\} \text{ in./s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= (-20\mathbf{i}) + (4\mathbf{k}) \times (4\mathbf{i} - 4\mathbf{k}) + (8\mathbf{j} - 3\mathbf{k})$$

$$\mathbf{v}_C = \{-20\mathbf{i} + 24\mathbf{j} - 3\mathbf{k}\} \text{ in./s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (-15\mathbf{i} - 80\mathbf{j}) + (3\mathbf{k}) \times (4\mathbf{i} - 4\mathbf{k}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (4\mathbf{i} - 4\mathbf{k})\right]$$
$$+ 2(4\mathbf{k}) \times (8\mathbf{j} - 3\mathbf{k}) + (-2.25\mathbf{i} + 44\mathbf{j} + 8\mathbf{k})$$

$$\mathbf{a}_C = \{-145\mathbf{i} - 24\mathbf{j} + 8\mathbf{k}\} \text{ in./s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{-20\mathbf{i} + 24\mathbf{j} - 3\mathbf{k}\}$ in./s
$\mathbf{a}_C = \{-145\mathbf{i} - 24\mathbf{j} + 8\mathbf{k}\}$ in./s^2

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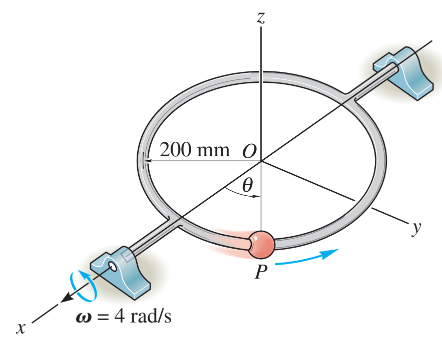
20–45.

The particle P slides around the circular hoop with a constant angular velocity of $\dot{\theta} = 6$ rad/s, while the hoop rotates about the x axis at a constant rate of $\omega = 4$ rad/s. If at the instant shown the hoop is in the x–y plane and the angle $\theta = 45°$, determine the velocity and acceleration of the particle at this instant.



SOLUTION

Relative to XYZ, let xyz have

$\Omega = \omega = \{4\mathbf{i}\}$ rad/s, $\quad \dot{\Omega} = \dot{\omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$\mathbf{r}_O = \mathbf{0}; \quad \mathbf{v}_O = \mathbf{0}; \quad \mathbf{a}_O = \mathbf{0}$

Relative to xyz, let coincident x', y', z', have

$\Omega_{xyz} = \{6\mathbf{k}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to XYZ.)

$(\mathbf{r}_{P/O})_{xyz} = 0.2\cos 45°\,\mathbf{i} + 0.2\sin 45°\,\mathbf{j} = \{0.1414\mathbf{i} + 0.1414\mathbf{j}\}$ m

$(\mathbf{r}_{P/O})_{xyz}$ ($(\mathbf{r}_{P/O})_{xyz}$ changes direction relative to XYZ.)

$$(\mathbf{v}_{P/O})_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{P/O}\right)_{xyz} = \mathbf{0} + (6\mathbf{k}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})$$

$$= \{-0.8485\mathbf{i} + 0.8485\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = \left(\ddot{\mathbf{r}}_{P/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'}\right] + \Omega \times \left(\mathbf{r}_{P/O}\right)_{xyz} + \Omega \times \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (6\mathbf{k}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) = \{-5.0912\mathbf{i} - 5.0912\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz} = \mathbf{0} + (4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j}) - 0.8485\mathbf{i} + 0.8485\mathbf{j}$$

$$= \{-0.849\mathbf{i} + 0.849\mathbf{j} + 0.566\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (4\mathbf{i}) \times \left[(4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})\right] + 2(4\mathbf{i}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) - 5.0912\mathbf{i} - 5.0912\mathbf{j}$$

$$= \{-5.09\mathbf{i} - 7.35\mathbf{j} + 6.79\mathbf{k}\} \text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_P = \{-0.849\mathbf{i} + 0.849\mathbf{j} + 0.566\mathbf{k}\}$ m/s
$\mathbf{a}_P = \{-5.09\mathbf{i} - 7.35\mathbf{j} + 6.79\mathbf{k}\}$ m/s^2

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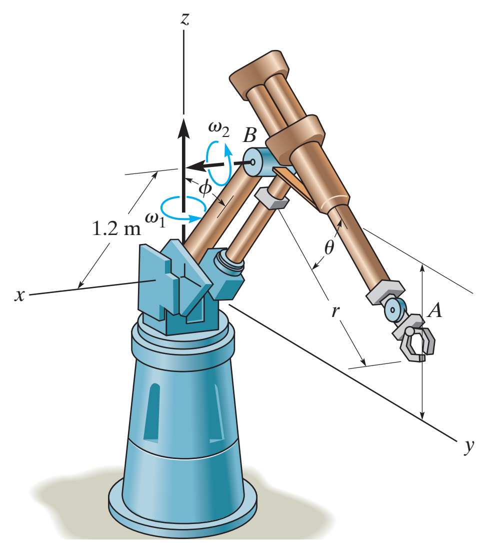
20–46.

At the instant shown, the industrial manipulator is rotating about the z axis at $\omega_1 = 5$ rad/s, and about joint B at $\omega_2 = 2$ rad/s. Determine the velocity and acceleration of the grip A at this instant, when $\phi = 30°$, $\theta = 45°$, and $r = 1.6$ m.



SOLUTION

$\Omega = \{5\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \mathbf{0}$

$\mathbf{r}_B = 1.2 \sin 30°\mathbf{j} + 1.2 \cos 30°\mathbf{k} = \{0.6\mathbf{j} + 1.0392\mathbf{k}\}$ m

$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = \mathbf{0} + (5\mathbf{k}) \times (0.6\mathbf{j} + 1.0392\mathbf{k}) = \{-3\mathbf{i}\}$ m/s

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$
$$= [\mathbf{0} + \mathbf{0}] + \mathbf{0} + [(5\mathbf{k}) \times (-3\mathbf{i})]$$
$$= \{-15\mathbf{j}\} \text{ m/s}^2$$

$\Omega_{xyz} = \{2\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \mathbf{0}$

$\mathbf{r}_{A/B} = 1.6 \cos 45°\mathbf{j} - 1.6 \sin 45°\mathbf{k} = \{1.1314\mathbf{j} - 1.1314\mathbf{k}\}$ m

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B}$$
$$= \mathbf{0} + (2\mathbf{i}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})$$
$$= \{2.2627\mathbf{j} + 2.2627\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \left[\dot{\Omega}_{xyz} \times \mathbf{r}_{A/B}\right] + \left[\Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}\right]$$

$$(\mathbf{a}_{A/B})_{xyz} = [\mathbf{0} + \mathbf{0}] + \mathbf{0} + \left[(2\mathbf{i}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})\right]$$
$$= \{-4.5255\mathbf{j} + 4.5255\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$
$$= (-3\mathbf{i}) + \left[(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})\right] + (2.2627\mathbf{j} + 2.2627\mathbf{k})$$
$$= \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$
$$= (-15\mathbf{j}) + \mathbf{0} + (5\mathbf{k}) \times [(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})] + [2(5\mathbf{k}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})] + (-4.5255\mathbf{j} + 4.5255\mathbf{k})$$
$$= \{-22.6\mathbf{i} - 47.8\mathbf{j} + 4.53\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}$ m/s
$\mathbf{a}_A = \{-22.6\mathbf{i} - 47.8\mathbf{j} + 45.3\mathbf{k}\}$ m/s^2

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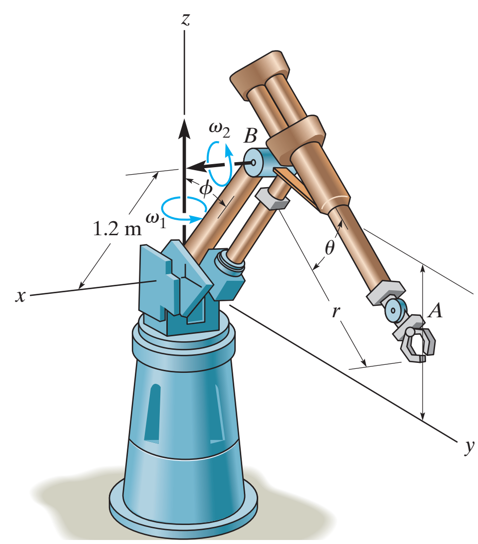
20–47.

At the instant shown, the industrial manipulator is rotating about the z axis at $\omega_1 = 5$ rad/s, and $\dot{\omega}_1 = 2$ rad/s^2; and about joint B at $\omega_2 = 2$ rad/s and $\dot{\omega}_2 = 3$ rad/s^2. Determine the velocity and acceleration of the grip A at this instant, when $\phi = 30°$, $\theta = 45°$, and $r = 1.6$ m.



SOLUTION

$$\boldsymbol{\Omega} = \{5\mathbf{k}\}\text{ rad/s} \qquad \dot{\boldsymbol{\Omega}} = \{2\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_B = 1.2\sin 30°\mathbf{j} + 1.2\cos 30°\mathbf{k} = \{0.6\mathbf{j} + 1.0392\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \boldsymbol{\Omega}\times\mathbf{r}_B = \mathbf{0} + (5\mathbf{k})\times(0.6\mathbf{j} + 1.0392\mathbf{k}) = \{-3\mathbf{i}\}\text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \boldsymbol{\Omega}\times(\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_B + \boldsymbol{\Omega}\times\dot{\mathbf{r}}_B$$

$$= [\mathbf{0} + \mathbf{0}] + (2\mathbf{k})\times(0.6\mathbf{j} + 1.0392\mathbf{k}) + [(5\mathbf{k})\times(-3\mathbf{i})]$$

$$= \{-1.2\mathbf{i} - 15\mathbf{j}\}\text{ m/s}^2$$

$$\boldsymbol{\Omega}_{xyz} = \{2\mathbf{i}\}\text{ rad/s} \qquad \dot{\boldsymbol{\Omega}}_{xyz} = \{3\mathbf{i}\}\text{ rad/s}^2$$

$$\mathbf{r}_{A/B} = 1.6\cos 45°\mathbf{j} - 1.6\sin 45°\mathbf{k} = \{1.1314\mathbf{j} - 1.1314\mathbf{k}\}\text{ m}$$

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \boldsymbol{\Omega}_{xyz}\times\mathbf{r}_{A/B}$$

$$= \mathbf{0} + (2\mathbf{i})\times(1.1314\mathbf{j} - 1.1314\mathbf{k})$$

$$= \{2.2627\mathbf{j} + 2.2627\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \boldsymbol{\Omega}_{xyz}\times(\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \left[\dot{\boldsymbol{\Omega}}_{xyz}\times\mathbf{r}_{A/B}\right] + \left[\boldsymbol{\Omega}_{xyz}\times\dot{\mathbf{r}}_{A/B}\right]$$

$$(\mathbf{a}_{A/B})_{xyz} = [\mathbf{0} + \mathbf{0}] + (3\mathbf{i})\times(1.1314\mathbf{j} - 1.1314\mathbf{k}) + [(2\mathbf{i})\times(2.2627\mathbf{j} + 2.2627\mathbf{k})]$$

$$= \{-1.1313\mathbf{j} + 7.9197\mathbf{k}\}\text{ m/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega}\times\mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$= (-3\mathbf{i}) + [(5\mathbf{k})\times(1.1314\mathbf{j} - 1.1314\mathbf{k})] + (2.2627\mathbf{j} + 2.2627\mathbf{k})$$

$$= \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}\text{ m/s}$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{A/B} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/B}) + 2\boldsymbol{\Omega}\times(\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= (-1.2\mathbf{i} - 15\mathbf{j}) + (2\mathbf{k})\times(1.1314\mathbf{j} - 1.1314\mathbf{k}) + (5\mathbf{k})\times[(5\mathbf{k})\times(1.1314\mathbf{j} - 1.1314\mathbf{k})] + [2(5\mathbf{k})\times(2.2627\mathbf{j} + 2.2627\mathbf{k})] + (-1.1313\mathbf{j} + 7.9197\mathbf{k})$$

$$= \{-26.1\mathbf{i} - 44.4\mathbf{j} + 7.92\mathbf{k}\}\text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}$ m/s
$\mathbf{a}_A = \{-26.1\mathbf{i} - 44.4\mathbf{j} + 7.92\mathbf{k}\}$ m/s^2

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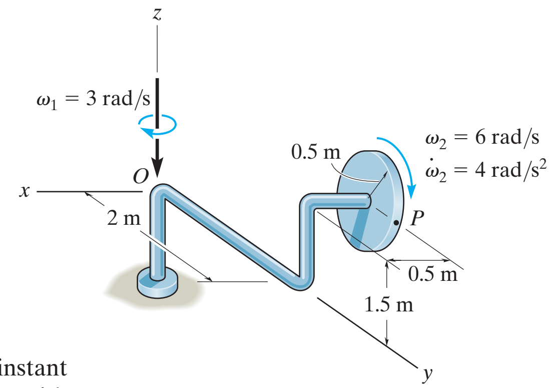
***20–48.**

At the given instant, the rod is turning about the z axis with a constant angular velocity $\omega_1 = 3$ rad/s. At this same instant, the disk is spinning at $\omega_2 = 6$ rad/s when $\dot{\omega}_2 = 4$ rad/s^2, both measured *relative* to the rod. Determine the velocity and acceleration of point P on the disk at this instant.



SOLUTION

Motion of point A. Point A is located at the center of the disk. At the instant consider, the fixed XYZ frame and rotating $x'\,y'\,z'$ frame are set coincident with origin at Point O. The $x'\,y'\,z'$ frame is set rotate with constant angular velocity of $\mathbf{\Omega}' = \boldsymbol{\omega}_1 = \{-3\mathbf{k}\}$ rad/s of which the direction does not change relative to XYZ frame. Since $\mathbf{\Omega}'$ is constant $\dot{\mathbf{\Omega}}' = \mathbf{0}$. Here $\mathbf{r}_A = \{-0.5\mathbf{i} + 2\mathbf{j} + 1.5\mathbf{k}\}$ m and its direction changes relative to XYZ frame but does not change relative to $x'y'z'$ frame. Thus,

$$\mathbf{v}_A = (\dot{r}_A)_{x'y'z'} + \mathbf{\Omega}' \times \mathbf{r}_A = \mathbf{0} + (-3\mathbf{k}) \times (-0.5\mathbf{i} + 2\mathbf{j} + 1.5\mathbf{k}) = \{6\mathbf{i} + 1.5\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_A = \left[(\ddot{r}_A)_{x'y'z'} + \mathbf{\Omega}' \times (\dot{r}_A)_{x'y'z'}\right] + \dot{\mathbf{\Omega}}' \times \mathbf{r}_A + \mathbf{\Omega}' \times \dot{\mathbf{r}}_A$$

$$= [\mathbf{0} + \mathbf{0}] + 0 + (-3\mathbf{k}) \times (6\mathbf{i} + 1.5\mathbf{j}) = \{4.5\mathbf{i} - 18\mathbf{j}\} \text{ m/s}^2$$

Motion of P with Respect to A. The xyz and $x''\,y''\,z''$ frame are set coincident with origin at A. Here $x''\,y''\,z''$ frame is set to rotate at $\mathbf{\Omega}'' = \boldsymbol{\omega}_z = \{-6\mathbf{i}\}$ rad/s of which its direction does not change relative to xyz frame. Also, $\Omega'' = \{-4\mathbf{i}\}$ rad/s^2. Here $(r_{P/A})_{xyz} = \{0.5\mathbf{j}\}$ m and its direction changes with respect to xyz frame but does not change relative to $x''\,y''\,z''$ frame.

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{r}_{P/A})_{xyz} = (\dot{r}_{P/A})_{x''y''z''} + \mathbf{\Omega} \times (r_{P/A})_{xyz}$$

$$= \mathbf{0} + (-6\mathbf{i}) \times (0.5\mathbf{j}) = \{-3\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = (\ddot{r}_{P/A})_{xyz} = \left[(\ddot{r}_{P/A})_{x''y''z''} + \Omega'' \times (\dot{r}_{P/A})_{x''y''z''}\right] + \dot{\mathbf{\Omega}}'' \times (\mathbf{r}_{P/A})_{xyz} + \Omega'' \times (\dot{r}_{P/A})_{xyz}$$

$$= [0 + \mathbf{0}] + (-4\mathbf{i}) \times (0.5\mathbf{j}) + (-6\mathbf{i}) \times (-3\mathbf{k})$$

$$= \{-18\mathbf{j} - 2\mathbf{k}\} \text{ m/s}^2$$

Motion of P. Here, $\mathbf{\Omega} = \boldsymbol{\omega}_1 = \{-3\mathbf{k}\}$ rad/s and $\dot{\mathbf{\Omega}} = \mathbf{0}$.

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$= (6\mathbf{i} + 1.5\mathbf{j}) + (-3\mathbf{k}) \times (0.5\mathbf{j}) + (-3\mathbf{k})$$

$$= \{7.50\mathbf{i} + 1.50\mathbf{j} - 3.00\mathbf{k}\} \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{P/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{P/A}) + 2\mathbf{\Omega} \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$= (4.5\mathbf{i} - 18\mathbf{j}) + 0 + (-3\mathbf{k}) \times (-3\mathbf{k} \times 0.5\mathbf{j}) + 2(-3\mathbf{k}) \times (-3\mathbf{k}) + (-18\mathbf{j} - 2\mathbf{k})$$

$$= \{4.50\mathbf{i} - 40.5\mathbf{j} - 2.00\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_P = \{7.50\mathbf{i} + 1.50\mathbf{j} - 3.00\mathbf{k}\}$ m/s
$\mathbf{a}_P = \{4.50\mathbf{i} - 40.5\mathbf{j} - 2.00\mathbf{k}\}$ m/s^2

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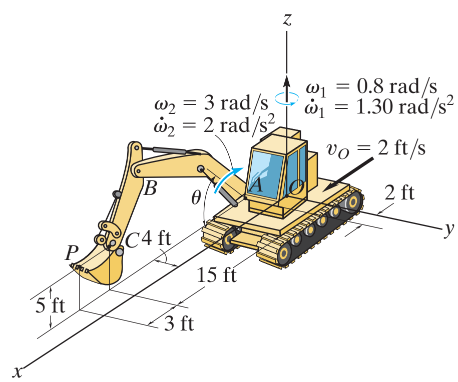
20–49.

At the instant shown, the backhoe is traveling forward at a constant speed $v_O = 2$ ft/s, and the boom ABC is rotating about the z axis with an angular velocity $\omega_1 = 0.8$ rad/s and an angular acceleration $\dot{\omega}_1 = 1.30$ rad/s^2. At this same instant the boom is rotating with $\omega_2 = 3$ rad/s when $\dot{\omega}_2 = 2$ rad/s^2, both measured relative to the frame. Determine the velocity and acceleration of point P on the bucket at this instant.



SOLUTION

$\mathbf{\Omega} = \{0.8\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = \{1.3\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = \{2\mathbf{i} - 4\mathbf{j}\}$ ft

$$\mathbf{v}_A = (\dot{\mathbf{r}}_A)_{xyz} + \mathbf{\Omega} \times \mathbf{r}_A$$
$$= (2\mathbf{i}) + (0.8\mathbf{k}) \times (2\mathbf{i} - 4\mathbf{j})$$
$$= \{5.20\mathbf{i} + 1.60\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_A = \left[(\ddot{\mathbf{r}}_A)_{xyz} + \mathbf{\Omega} \times (\dot{\mathbf{r}}_A)_{xyz}\right] + \dot{\mathbf{\Omega}} \times \mathbf{r}_A + \mathbf{\Omega} \times \dot{\mathbf{r}}_A$$
$$= \mathbf{0} + (0.8\mathbf{k}) \times (2\mathbf{i}) + (1.3\mathbf{k}) \times (2\mathbf{i} - 4\mathbf{j}) + (0.8\mathbf{k}) \times (5.20\mathbf{i} + 1.60\mathbf{j})$$
$$= \{3.92\mathbf{i} + 8.36\mathbf{j}\} \text{ ft/s}^2$$

$\mathbf{\Omega}_{P/A} = \{-3\mathbf{j}\}$ rad/s

$\dot{\mathbf{\Omega}}_{P/A} = \{-2\mathbf{j}\}$ rad/s^2

$\mathbf{r}_{P/A} = \{16\mathbf{i} + 5\mathbf{k}\}$ ft

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{xyz} + \mathbf{\Omega}_{P/A} \times \mathbf{r}_{P/A}$$
$$= \mathbf{0} + (-3\mathbf{j}) \times (16\mathbf{i} + 5\mathbf{k})$$
$$= \{-15\mathbf{i} + 48\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/A})_{xyz} + \mathbf{\Omega}_{P/A} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right] + \dot{\mathbf{\Omega}}_{P/A} \times \mathbf{r}_{P/A} + \mathbf{\Omega}_{P/A} \times \dot{\mathbf{r}}_{P/A}$$
$$= \mathbf{0} + \mathbf{0} + (-2\mathbf{j}) \times (16\mathbf{i} + 5\mathbf{k}) + (-3\mathbf{j}) \times (-15\mathbf{i} + 48\mathbf{k})$$
$$= \{-154\mathbf{i} - 13\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$
$$= (5.2\mathbf{i} + 1.6\mathbf{j}) + (0.8\mathbf{k}) \times (16\mathbf{i} + 5\mathbf{k}) + (-15\mathbf{i} + 48\mathbf{k})$$

$\mathbf{v}_P = \{-9.80\mathbf{i} + 14.4\mathbf{j} + 48.0\mathbf{k}\}$ ft/s **Ans.**

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{P/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{P/A}) + 2\mathbf{\Omega} \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$
$$= (3.92\mathbf{i} + 8.36\mathbf{j}) + (1.3\mathbf{k}) \times (16\mathbf{i} + 5\mathbf{k}) + (0.8\mathbf{k}) \times [(0.8\mathbf{k}) \times (16\mathbf{i} + 5\mathbf{k})]$$
$$+ 2(0.8\mathbf{k}) \times (-15\mathbf{i} + 48\mathbf{k}) + (-154\mathbf{i} - 13\mathbf{j})$$

$\mathbf{a}_P = \{-160\mathbf{i} + 5.16\mathbf{j} - 13\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_P = \{-9.80\mathbf{i} + 14.4\mathbf{j} + 48.0\mathbf{k}\}$ ft/s
$\mathbf{a}_P = \{-160\mathbf{i} + 5.16\mathbf{j} - 13\mathbf{k}\}$ ft/s^2

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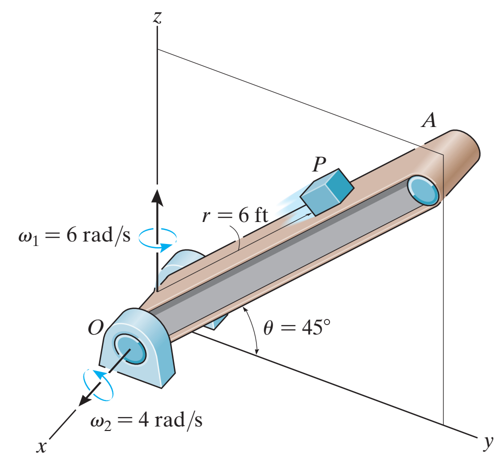
20–50.

At the instant shown, the arm *OA* of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a constant rate $\dot{r} = 5$ ft/s, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



SOLUTION

$\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$

$\Omega_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega}_{P/O} = \mathbf{0}$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$$

$$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$$

$$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}\text{ ft/s}$$

$$(\mathbf{a}_{P/O}) = (\ddot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times (\dot{\mathbf{r}}_{P/O})_{xyz} + \dot{\Omega}_{P/O} \times \mathbf{r}_{P/O} + \Omega_{P/O} \times \dot{\mathbf{r}}_{P/O}$$

$$= \mathbf{0} + (4\mathbf{i}) \times (3.536\mathbf{j} + 3.536\mathbf{k}) + \mathbf{0} + (4\mathbf{i}) \times (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$$= \{-96.18\mathbf{j} - 39.60\mathbf{k}\}\text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{r}_{P/O})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans.**

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-96.18\mathbf{j} - 39.60\mathbf{k})$$

$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s
$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}$ ft/s^2

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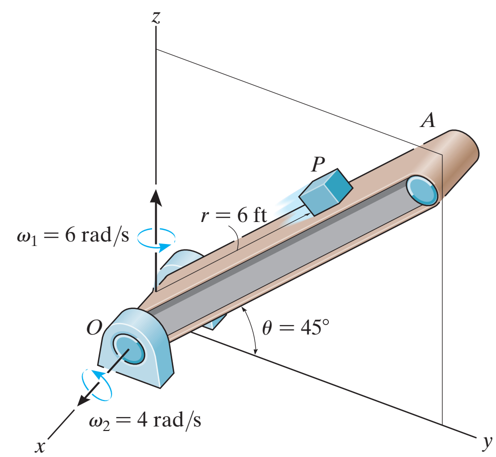
20–51.

At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a rate $\dot{r} = 5$ ft/s, which is increasing at $\ddot{r} = 8$ ft/s^2, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



SOLUTION

$\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$

$\Omega_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega}_{P/O} = \mathbf{0}$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$

$= (5 \cos 45°\mathbf{j} + 5 \sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$

$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}$ ft/s

$(\mathbf{a}_{P/O})_{xyz} = 8 \cos 45\mathbf{j} + 8 \sin 45°\mathbf{k} - 96.18\mathbf{j} - 39.60\mathbf{k}$

$= \{-90.52\mathbf{j} - 33.945\mathbf{k}\}$ ft/s^2

$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$

$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$

$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans.**

$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$

$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-90.52\mathbf{j} - 33.945\mathbf{k})$

$= -152.75\mathbf{j} + 161.23\mathbf{i} - 90.52\mathbf{j} - 33.945\mathbf{k}$

$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s
$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\}$ ft/s^2

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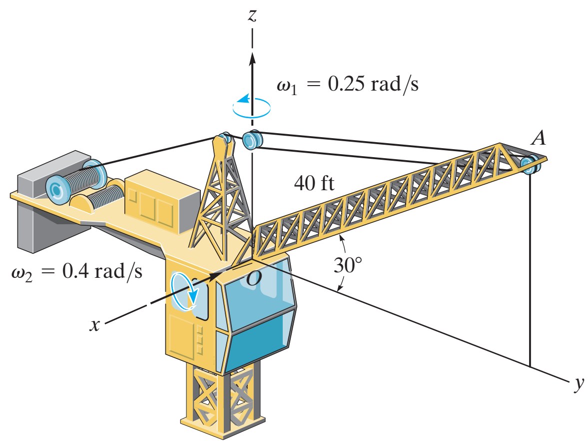
***20–52.**

The crane is rotating about the z axis with a constant rate $\omega_1 = 0.25$ rad/s, while the boom OA is rotating downward with a constant rate $\omega_2 = 0.4$ rad/s. Compute the velocity and acceleration of point A located at the top of the boom at the instant shown.



SOLUTION

$\mathbf{\Omega} = \{0.25\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = 0$

$\mathbf{r}_O = 0$

$\mathbf{v}_O = 0$

$\mathbf{a}_O = 0$

$\mathbf{\Omega}_{A/O} = \{-0.4\mathbf{i}\}$ rad/s

$\dot{\mathbf{\Omega}}_{A/O} = 0$

$$\mathbf{r}_{A/O} = 4\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$$
$$= 34.64\mathbf{j} + 20\mathbf{k}$$

$$(\mathbf{v}_{A/O})_{xyz} = (\dot{\mathbf{r}}_{A/O})_{xyz} + \mathbf{\Omega}_{A/O} \times \mathbf{r}_{A/O}$$
$$= \mathbf{0} + (-0.4\mathbf{i}) \times (34.64\mathbf{j} + 20\mathbf{k})$$
$$= 8\mathbf{j} - 13.856\mathbf{k}$$

$$(\mathbf{a}_{A/O})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/O})_{xyz} + \mathbf{\Omega}_{A/O} \times (\dot{\mathbf{r}}_{A/O})_{xyz}\right] + \dot{\mathbf{\Omega}}_{A/O} \times \mathbf{r}_{A/O} + \mathbf{\Omega}_{A/O} \times \dot{\mathbf{r}}_{A/O}$$
$$= 0 + 0 + 0 + (-4\mathbf{i}) \times (8\mathbf{j} - 13.86\mathbf{k})$$
$$= -5.542\mathbf{j} - 3.2\mathbf{k}$$

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$$
$$= 0 + 0.25\mathbf{k} \times (34.64\mathbf{j} + 20\mathbf{k}) + (8\mathbf{j} - 13.856\mathbf{k})$$
$$= \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}\ \text{ft/s}$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{A/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$
$$= 0 + 0 + (0.25\mathbf{k}) \times (0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$$
$$+ 2(0.25\mathbf{k}) \times (8\mathbf{j} - 13.856\mathbf{k}) - 5.542\mathbf{j} - 3.2\mathbf{k}$$
$$\mathbf{a}_A = \{-4\mathbf{i} - 7.71\mathbf{j} - 3.20\mathbf{k}\}\ \text{ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-4\mathbf{i} - 7.71\mathbf{j} - 3.20\mathbf{k}\}$ ft/s^2

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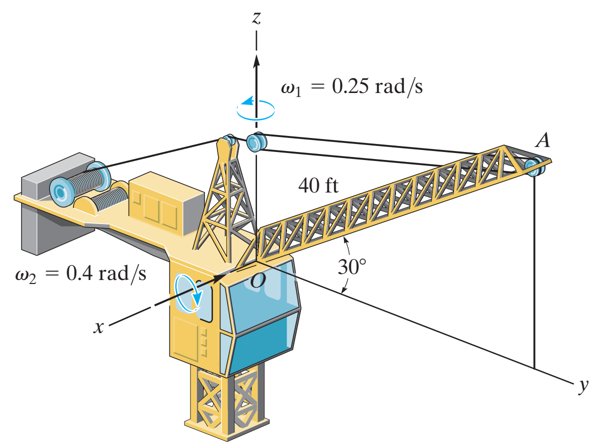
20–53.

Solve Prob. 20–52 if the angular motions are increasing at $\dot{\omega}_1 = 0.4\ \text{rad/s}^2$ and $\dot{\omega}_2 = 0.8\ \text{rad/s}^2$ at the instant shown.



SOLUTION

$\mathbf{\Omega} = \{0.25\mathbf{k}\}\ \text{rad/s}$

$\dot{\mathbf{\Omega}} = \{0.4\mathbf{k}\}\ \text{rad/s}^2$

$\mathbf{r}_O = 0$

$\mathbf{v}_O = 0$

$\mathbf{a}_O = 0$

$\mathbf{\Omega}_{A/O} = \{-0.4\mathbf{i}\}\ \text{rad/s}$

$\dot{\mathbf{\Omega}}_{A/O} = \{-0.8\mathbf{i}\}\ \text{rad/s}^2$

$\mathbf{r}_{A/O} = 4\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$

$\quad = 34.64\mathbf{j} + 20\mathbf{k}$

$(\mathbf{v}_{A/O})_{xyz} = (\dot{\mathbf{r}}_{A/O})_{xyz} + \Omega_{A/O} \times \mathbf{r}_{A/O}$

$\quad = 0 + (-0.4\mathbf{i}) \times (34.64\mathbf{j} + 20\mathbf{k})$

$\quad = 8\mathbf{j} - 13.856\mathbf{k}$

$(\mathbf{a}_{A/O})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/O})_{xyz} + \mathbf{\Omega}_{A/O} \times (\dot{\mathbf{r}}_{A/O})_{xyz}\right] + \dot{\mathbf{\Omega}}_{A/O} \times \mathbf{r}_{A/O} + \mathbf{\Omega}_{A/O} \times \dot{\mathbf{r}}_{A/O}$

$\quad = 0 + 0 + (-0.8\mathbf{i}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-4\mathbf{i}) \times (8\mathbf{j} - 13.86\mathbf{k})$

$\quad = 10.457\mathbf{j} - 30.913\mathbf{k}$

$\mathbf{v}_A = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$

$\quad = 0 + 0.25\mathbf{k} \times (34.64\mathbf{j} + 20\mathbf{k}) + (8\mathbf{j} - 13.856\mathbf{k})$

$\quad = \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}\ \text{ft/s}$ **Ans.**

$\mathbf{a}_A = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{A/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$

$\quad = 0 + (0.4\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$
$\quad + (0.25\mathbf{k}) \times [(0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})] + 2(0.25\mathbf{k}) \times (8\mathbf{j} - 13.856\mathbf{k}) + 10.457\mathbf{j} - 30.913\mathbf{k}$

$\mathbf{a}_A = \{-17.9\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}\ \text{ft/s}^2$ **Ans.**

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}\ \text{ft/s}$
$\mathbf{a}_A = \{-17.9\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}\ \text{ft/s}^2$

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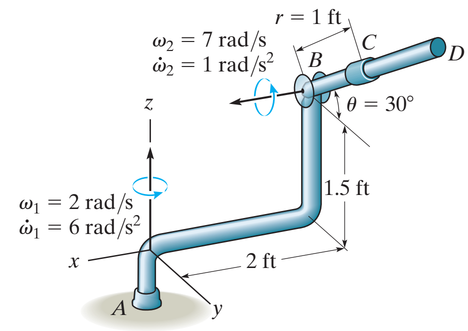
20–54.

At the instant shown, the arm AB is rotating about the fixed bearing with an angular velocity $\omega_1 = 2$ rad/s and angular acceleration $\dot{\omega}_1 = 6$ rad/s^2. At the same instant, rod BD is rotating relative to rod AB at $\omega_2 = 7$ rad/s, which is increasing at $\dot{\omega}_2 = 1$ rad/s^2. Also, the collar C is moving along rod BD with a velocity $\dot{r} = 2$ ft/s and a *deceleration* $\ddot{r} = -0.5$ ft/s^2, both measured relative to the rod. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

$$\mathbf{\Omega} = \{2\mathbf{k}\} \text{ rad/s}$$

$$\dot{\mathbf{\Omega}} = \{6\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_B = \{-2\mathbf{i} + 1.5\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \mathbf{\Omega} \times \mathbf{r}_B$$

$$= (2\mathbf{k}) \times (-2\mathbf{i} + 1.5\mathbf{k})$$

$$= \{-4\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \mathbf{\Omega} \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\mathbf{\Omega}} \times \mathbf{r}_B + \mathbf{\Omega} \times \dot{\mathbf{r}}_B$$

$$= (6\mathbf{k}) \times (-2\mathbf{i} + 1.5\mathbf{k}) + (2\mathbf{k}) \times (-4\mathbf{j})$$

$$= \{8\mathbf{i} - 12\mathbf{j}\} \text{ ft/s}^2$$

$$\mathbf{\Omega}_{C/B} = \{7\mathbf{i}\} \text{ rad/s}$$

$$\dot{\mathbf{\Omega}}_{C/B} = \{1\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_{C/B} = 1 \cos 30°\mathbf{j} + 1 \sin 30°\mathbf{k} = \{0.866\mathbf{j} + 0.5\mathbf{k}\} \text{ ft}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} + \mathbf{\Omega}_{C/B} \times \mathbf{r}_{C/B}$$

$$= (2 \cos 30°\mathbf{j} + 2 \sin 30°\mathbf{k}) + (7\mathbf{i}) \times (0.866\mathbf{j} + 0.5\mathbf{k})$$

$$= \{-1.768\mathbf{j} + 7.062\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{xyz} + \mathbf{\Omega}_{C/B} \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right] + \dot{\mathbf{\Omega}}_{C/B} \times \mathbf{r}_{C/B} + \mathbf{\Omega}_{C/B} \times \dot{\mathbf{r}}_{C/B}$$

$$= (-0.5 \cos 30°\mathbf{j} - 0.5 \sin 30°\mathbf{k}) + (7\mathbf{i}) \times (1.732\mathbf{j} + 1\mathbf{k}) + (1\mathbf{i}) \times (0.866\mathbf{j} + 0.5\mathbf{k}) + (7\mathbf{i}) \times (-1.768\mathbf{j} + 7.06\mathbf{k})$$

$$= \{-57.37\mathbf{j} + 0.3640\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= (-4\mathbf{j}) + (2\mathbf{k}) \times (0.866\mathbf{j} + 0.5\mathbf{k}) + (-1.768\mathbf{j} + 7.06\mathbf{k})$$

$$\mathbf{v}_C = \{-1.73\mathbf{i} - 5.77\mathbf{j} + 7.06\mathbf{k}\} \text{ ft/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/B} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/B}) + 2\mathbf{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (8\mathbf{i} - 12\mathbf{j}) + (6\mathbf{k}) \times (0.866\mathbf{j} + 0.5\mathbf{k}) + (2\mathbf{k}) \times \left[(2\mathbf{k}) \times (0.866\mathbf{j} + 0.5\mathbf{k})\right] + 2(2\mathbf{k}) \times (-1.768\mathbf{j} + 7.062\mathbf{k}) + (-57.37\mathbf{j} + 0.364\mathbf{k})$$

$$\mathbf{a}_C = \{9.88\mathbf{i} - 72.8\mathbf{j} + 0.365\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{-1.73\mathbf{i} - 5.77\mathbf{j} + 7.06\mathbf{k}\}$ ft/s
$\mathbf{a}_C = \{9.88\mathbf{i} - 72.8\mathbf{j} + 0.365\mathbf{k}\}$ ft/s^2